Lecture 3

Measurement (3)

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Uncertainty analysis

an accurate measurement is one that is close to the 'true answer'. However, in practice we do not know what the 'true answer' is. In the real world, what interests us is the answer to the question:

"How wrong are we likely to have been?"

The answer to this question is called the 'uncertainty of measurement'

In short, we are looking to identify the possible sources of uncertainty, evaluate the uncertainty from each source and, finally, combine the individual uncertainties to get an overall figure.

Uncertainty analysis

The eight main steps to evaluating uncertainty	
1	Decide what you need to find from your measurements. Decide what actual measurements and calculations are needed to produce the final result.
2	Carry out the measurements needed.
3	Evaluate the uncertainty of each input quantity that feeds in to the final result Express all uncertainties in similar terms (standard uncertainties).
4	Decide whether the errors of the input quantities are independent of each other.
5	Calculate the result of your measurement (including any known corrections for things such as calibrations).
6	Find the combined standard uncertainty from all the individual aspects.
7	Express the uncertainty in terms of a coverage factor together with an expanded uncertainty at a stated level of confidence.
8	Record the measurement result and the uncertainty, and state how you got both of these.

Uncertainty analysis

1. Decide what you need to find out from your measurement

Identify the type of measurement and how it is to be measured, as well as any calculations required in the process such as effects that require a correction.

Uncertainty analysis

Example

suppose you decide to use a set of electronic Vernier caliper to measure the length of an object.

Uncertainty analysis

2. Carry out and record the measurements needed

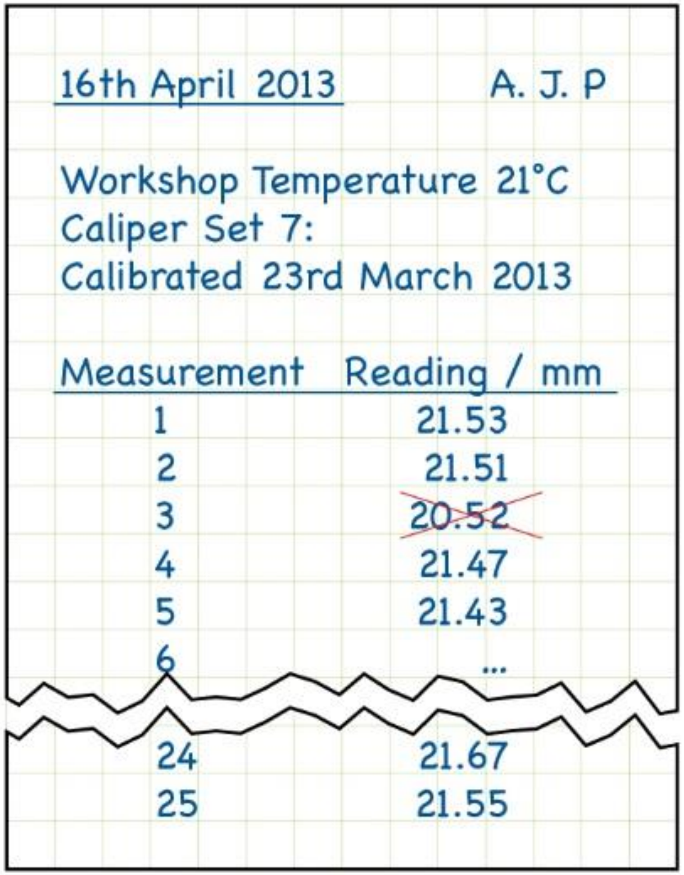
Following a specified measurement procedure to ensure that your measurement is consistent with that of other colleagues in your organization.

Uncertainty analysis

Example

know they are well maintained and calibrated checked the zero reading on your electronic calipers then you took repeated readings.

Notice how clearly you have set out your notebook, with a date, initials, the instruments you are using, a note of the calibration sticker on the callipers, and a record of the temperature.



Average : (21.53 + 21.51 + 21.47 + 21.43 +…) ÷ 24 = 21.493 mm

Uncertainty analysis

3. Evaluate the uncertainty of each input quantity that feeds in to the final result. Express all uncertainties in similar terms (standard uncertainties)

How wrong is this result likely to be? What factors could have affected your measurement?

1. **Type A** uncertainty evaluations are carried out by statistical methods, usually from repeated measurement readings.
2. **Type B** uncertainty evaluations are carried out using any other information such as past experiences, calibration certificates, manufacturers specifications, from calculation, from published information and from common sense.

Uncertainty analysis

Type A uncertainty evaluation

Type A uncertainty evaluations are carried out by statistical methods, usually from repeated measurement readings.

characterize the variability of n readings by their standard deviation, given by the formula below

$$\text{standard deviation} = \sqrt{\frac{\sum_{i=1}^{n}\left(\text{reading}_i - \text{average}\right)^2}{n-1}}$$

Uncertainty analysis

Type A uncertainty evaluation

Taking more readings would improve your confidence in the estimate of the average.

For normal probability distribution

$$\text{standard uncertainty} = \frac{\text{standard deviation}}{\sqrt{n}}$$

Uncertainty analysis

Example

You have 24 readings and have used these to gain an average of 21.493 mm.

Measurement	Reading / mm	(Reading-Average)	(Reading-Average)
1	21.53	0.0375	0.0014
2	21.51	0.0175	0.0003
3	~~20.52~~		
4	21.47	-0.0225	0.0005
5	21.43	-0.0625	0.0039
6	...		...
...	...	...	...
25	21.55	0.0575	0.0033
Average	21.493	Sum	0.2505
		(Sum/23)	0.0109

Standard Deviation $=\sqrt{(Sum/23)} = 0.1044$

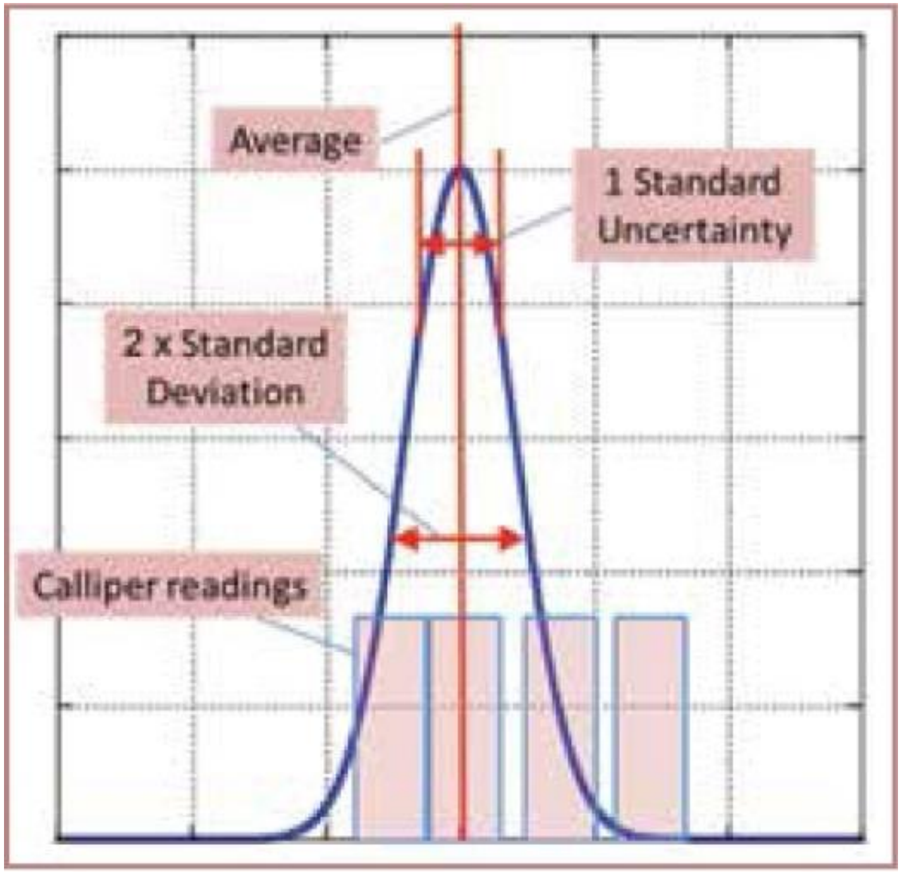
Uncertainty analysis

Example

standard uncertainty

$$\frac{0.1044 \text{ mm}}{\sqrt{24}} = 0.021 \text{ mm}$$

This uncertainty is based upon the idea that the readings you took were drawn from a normal probability distribution. You used your 24 readings to estimate the characteristics of this distribution – and then worked out the standard uncertainty – how well you can estimate the position of the center of the distribution.

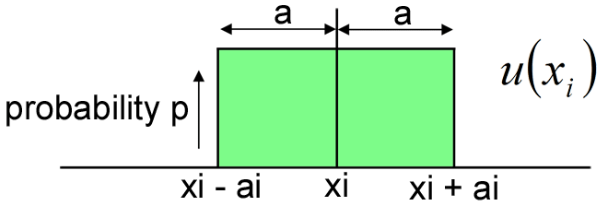


Calliper reading

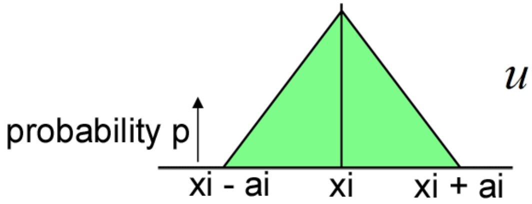
Uncertainty analysis

Type B uncertainty evaluation

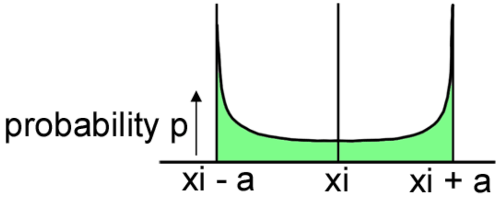
Type B uncertainty evaluations are carried out using any other information such as past experiences, calibration certificates, manufacturers' specifications, from calculation, from published information and from common sense.



$$u(x_i) = \frac{a_i}{\sqrt{3}}$$



$$u(x_i) = \frac{a_i}{\sqrt{6}}$$



$$u(x_i) = \frac{a_i}{\sqrt{2}}$$

$$\text{standard uncertainty} = \frac{\text{half range of deviation}}{\sqrt{3}}$$

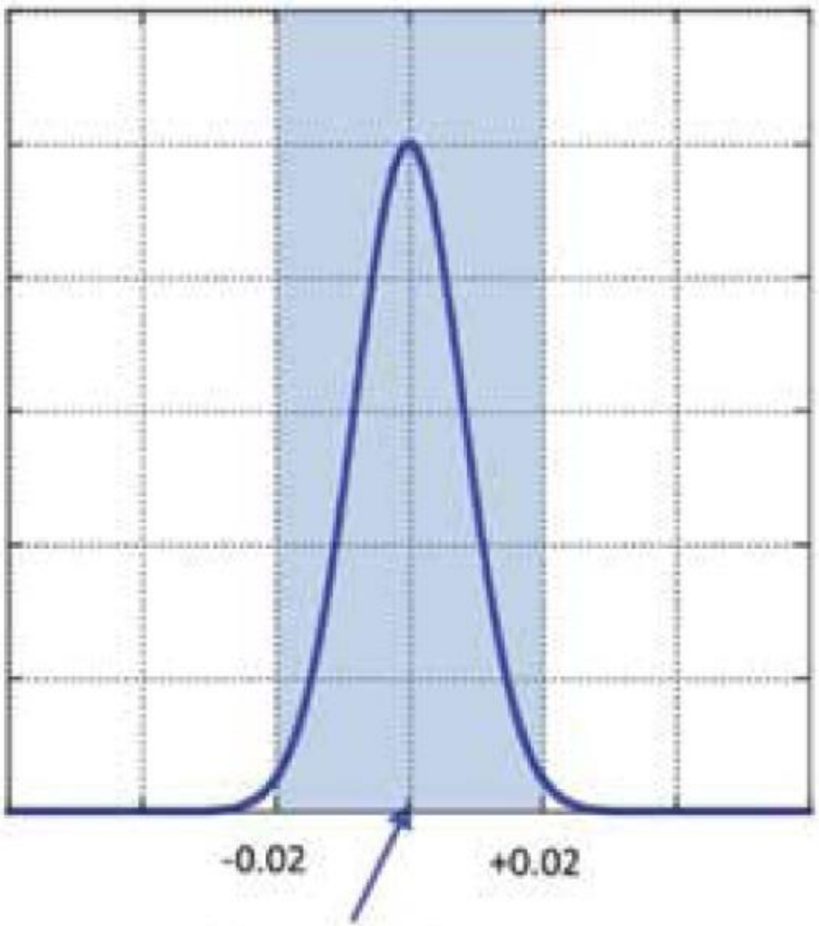
Uncertainty analysis

Example

In this example, the calibration certificate simply states that the device will read within ±0.02 mm of the correct value, if it is used correctly and the temperature is within the range 0 °C to 40 °C.

The standard uncertainty associated with the calibration of the device is thus :

$$0.02\ \text{mm}/\sqrt{3} = 0.012\ \text{mm}.$$



Uncertainty analysis

4. Decide whether the errors of the input quantities are independent of each other

Could a large error in one input cause a large error in another?

Could an outside influence such as temperature have a similar effect on several aspects of uncertainty at once?

Uncertainty analysis

Example

Assuming that there is no correlation can lead to an unreliable uncertainty evaluation.

If the errors are independent, which is typical and assumed in this example, you can use the formula in step 6 to calculate combined standard uncertainty. If not, extra calculations are needed, beyond this guide.

Uncertainty analysis

5. Calculate the result of your measurement (including any known corrections, such as calibrations)

You get your result from the mean reading and by making all necessary corrections to it, such as calibration corrections listed on a calibration certificate.

Uncertainty analysis

Example

In this example you do not have any certificate corrections to include. But if you did, they would be added to the original mean reading of 21.493 mm.

Uncertainty analysis

6. Find the combined standard uncertainty from all the individual uncertainty contributions

Once you have your individual standard uncertainties they need to be combined. But how do you combine the Type A and Type B evaluation of uncertainty? You could simply add the two numbers, but that would give a pessimistic assessment of the uncertainty because it is unlikely that both factors would be at the limit of their range. So in order to evaluate the uncertainty you add the components 'in quadrature' (also known as 'the root sum of the squares'). The result of this is called the 'combined standard uncertainty'.

$$\text{overall uncertainty} = \sqrt{(\text{component}_1)^2 + (\text{component}_2)^2}$$

Uncertainty analysis

Example

$$\text{overall uncertainty} = \sqrt{(0.021)^2 + (0.012)^2} = 0.024 \text{ mm}$$

Uncertainty analysis

7. Calculate expanded uncertainty for a particular level of confidence

The combined standard uncertainty may be thought of as equivalent to one standard deviation, the mean ± one standard deviation covers about 68 % of the normal distribution.

You can increase the level of confidence that the true answer lies within a given a range, by multiplying the standard uncertainty by a coverage factor to give an expanded uncertainty.

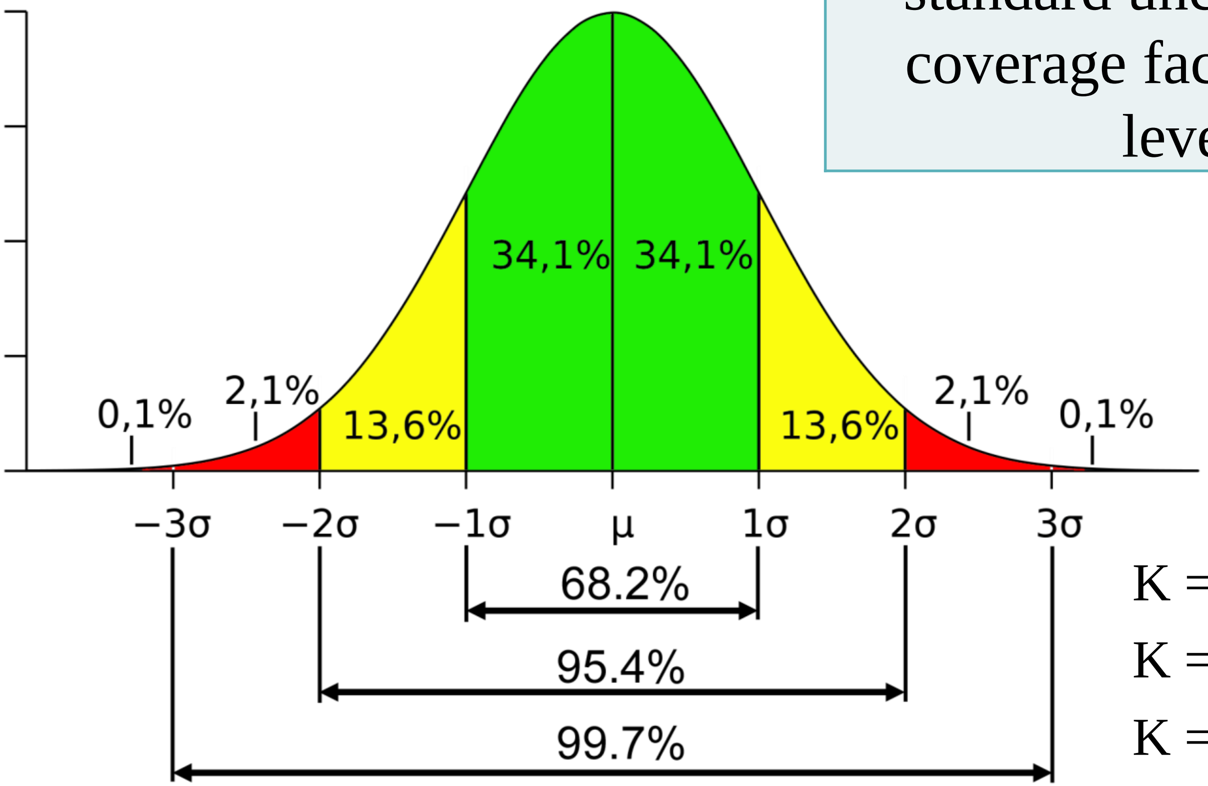
Expanded uncertainty U = coverage factor k × combined standard uncertainty.

You can increase the confidence level to 95 % or even 99 % by combining with the appropriate coverage factor (assuming a normal distribution).

Uncertainty analysis

Expanded uncertainty

Is the standard uncertainty (or combined standard uncertainty) multiplied by a coverage factor k to give a particular level of confidence



K =1 ⇔ confidence level 68.2%

K =2 ⇔ confidence level 95.4%

K =3 ⇔ confidence level 99.7%

Uncertainty analysis

Example

Standard Uncertainty	Coverage Factor	Expanded Uncertainty	Probability that true value lies in range
0.024 mm	1	0.024 mm	68.2 %
	2	0.048 mm	95.4 %
	3	0.072 mm	99.7 %

Uncertainty analysis

8. Write down the measurement result and the uncertainty, and state how you got both of these

It is important to express the result so that a reader can use the information.

The main things to mention are:

1. The measurement result, together with the uncertainty
2. The statement of the coverage factor and the level of confidence.
3. A recommended wording is: "The reported uncertainty is based on a standard uncertainty multiplied by coverage factor k = 2, providing a level of confidence of approximately 95 %."
4. How the uncertainty was evaluated.

Uncertainty analysis

Example

In this example, you write:

21.493 ± 0.048 mm

Source of Uncertainty	Value/mm	Probability Distribution	Factor	Standard Uncertainty/mm
Variability	0.021	Normal	1	0.021
Calibration	± 0.02	Rectangular	$1/\sqrt{3}$	0.012
Standard Uncertainty				0.024
Expanded Uncertainty (coverage factor 2)				0.048