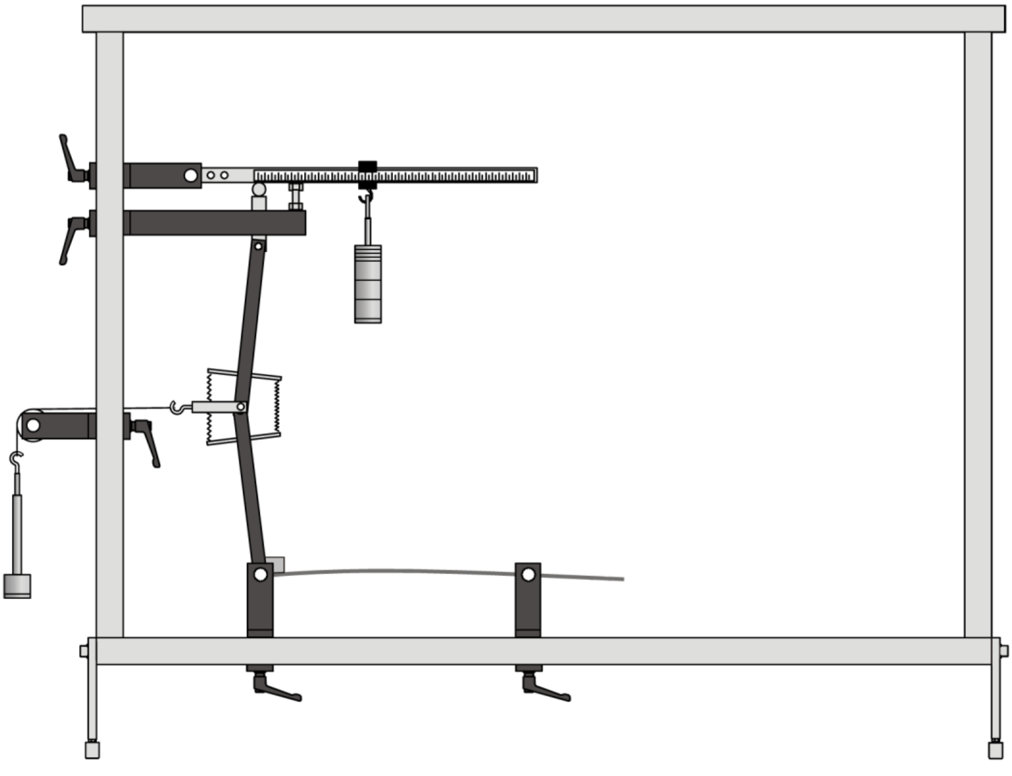
Laboratory 3

Investigation of Simple Stability Problem



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Contents

1. Objectives:

Learning Objectives

- Determination of the buckling force for the case of an:
  - elastic joint
  - elastic fixed end support
- Investigation of the buckling behavior under the influence of:
  - of additional shear forces
  - of pre-deformation

2. Introduction:

Investigation of Simple Stability Problems device is designed to be used in conjunction with the Mounting Frame.

The trainer enables experimental investigation simple stability problems. The central piece is a buckling bar with center joint.

3. Equipment description:

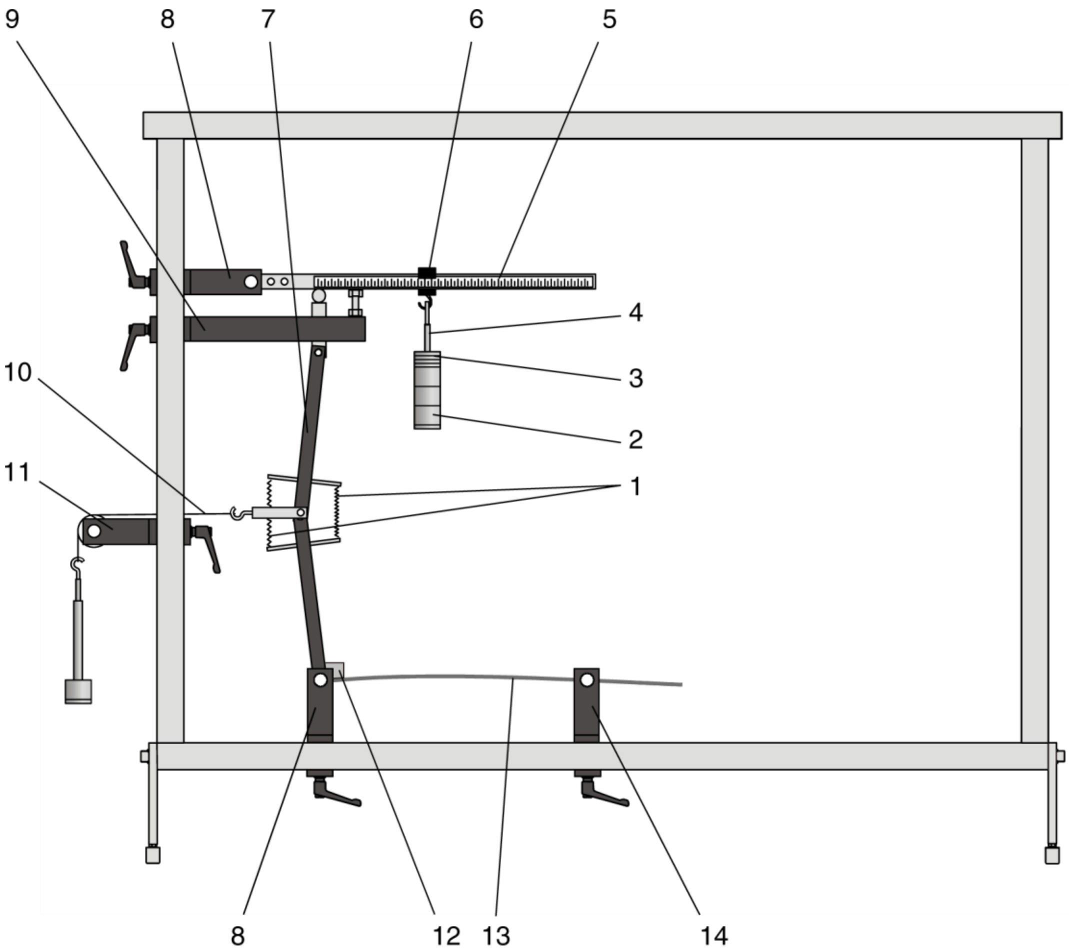

Fig. 3.1 Loading frame

1	Tension spring
2	Weight 5 N
3	Weight 1 N
4	Weights hanger, own weight 1 N
5	Loading lever with scale
6	Sliding rider, movable
7	Kink bar with joint
8	Joint bearing
9	Upper bearing with compression piece and stop screw

10 Traction rope

11 Joint bearing with pulley

12 Angle piece for leaf spring

13 Leaf spring

14 Joint bearing with clamping bracket

This device has the following features:

- ✓ Infinite adjustment of loading by means of lever and weight set.
- ✓ Precise determination of loading by means of accurate scale on loading lever.
- ✓ Different degrees of loading are possible by means of additional spiral spring with variable length on lower bearing.
- ✓ Device for generation lateral force.
- ✓ Compression piece inserted without friction in ball socket.
- ✓ Frictionless joint with rolling bearing.

4. Formula Symbols and Units Used

Symbol	Mathematical/physical quantity	Unit
a , b	Length	mm
k	Spring constant	Nm
k_φ	Torsional stiffness	Nm / rad
E	Elasticity modulus	N / mm^2
F_1 , F_2	Tension spring forces	N
F_j	Cutting force at joint	N
F_k	Buckling force	N
F_V	Lever‘s own weight, compression piece and buckling bar	N
G	Weights and weight hanger	N
I	Moment of area	mm^4
L	Length bar part	mm
M_a	Moment at bearing point	Nm
M_j	Moment at joint	Nm

5. Coefficients and specimen's characteristics

Compressive force	
Load due to own weight	$21.7\,N$
Buckling rod	
Length	$2\times 250\ mm$
Elastic joint	
Tension spring constant	$k = 2\ \ N/mm$
Lever arm Length	$50\ mm$
Leaf spring:	
Area moment of inertia	$6.66\ mm^4$
Material is steel with Elasticity modulus	$205000\ \ N/mm^2$

6. Safety Instructions

ATTENTION	ATTENTION Respect the load mentioned in the procedure

7. Basic principles

The basic principles set out in the following make no claim to completeness. For further theoretical explanations, refer to the specialist literature.

A system is considered to be stable if it returns to its original position after being displaced from the equilibrium position, e.g. by an external force.

A particularly simple model for a system, which can be unstable, is the **buckling bar with elastic joint** (Fig. 3.2). This is stable until a certain loading condition. When this limit load (buckling force F_k) is exceeded, where the system buckles suddenly, it will be unstable.

The elastic joint is facilitated by means of placing a turning spring between the two bar parts with torsional stiffness k_φ.

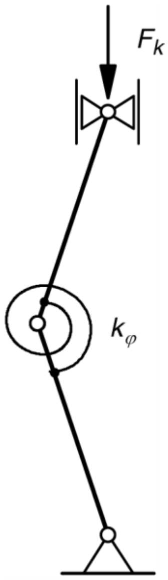

Fig. 3.2 Buckling bar with elastic joint

This spring will be realized in this training kit by using two tension springs (force constant k) on two levers (length a). First the torsional stiffness will be determined. Since the displacement is going to be small, the angle relation is linear:

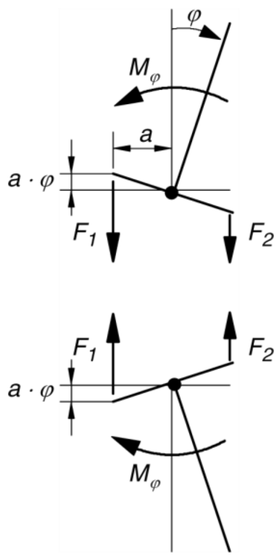

Fig. 3.3

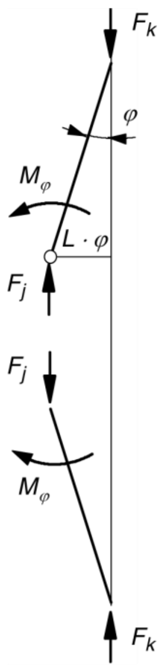

Fig. 3.4

$\sin\varphi = \varphi$ Eq. 3.1

$\cos\varphi = 1$ Eq. 3.2

$M_\varphi = k_\varphi.\varphi = (F_1 - F_2).a$ Eq. 3.3

or

$k_\varphi = (F_1 - F_2).\dfrac{a}{\varphi}$ Eq. 3.4

Springs forces are determined form the stiffness and the spring displacement

$F_1 = 2.k.a.\varphi$ Eq. 3.5

And

$F_2 = -2.k.a.\varphi$ Eq. 3.6

From which the torsional stiffness is

$k_\varphi = 4.k.a^2$ Eq. 3.7

For stability the part bars must be in equilibrium.

The moment equilibrium about the joint applies for the free cut of the upper bar part.

$\sum M_J = 0 = M_\varphi - F_k.L.\varphi = k_\varphi.\varphi - F_k.L.\varphi$ Eq. 3.8

At stability, this equilibrium condition must apply for non-zero displacement.

$k_\varphi.\varphi - F_k.L.\varphi = \varphi\left(4.k.a^2 - F_k.L\right)$ Eq. 3.9

from which the buckling force results.

$F_k = \dfrac{4.k.a^2}{L}$ Eq. 3.10

When the compression force exceeds the buckling force, and therefore the system is not stable, the bar buckles. If the compression force is equal to the buckling force, the system is stable and returns to its original position after the effect of an external force.

Fig. 3.5

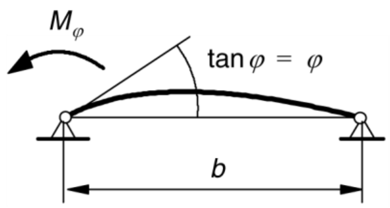

Fig. 3.6

In the case of an elastic bearing, the procedure is similar. The elastic bearing is realized by means of a leaf spring coupled into the lower joint in the training kit.

In case of a side moment and small angles, the leaf spring (length b) obeys the following relation:

$$\tan\varphi = \varphi = \frac{M_\varphi . b}{3.E.\mathrm{I}} \qquad \text{Eq. 3.11}$$

Thus the tensile stiffness is

$$k_\varphi = \frac{3.E.\mathrm{I}}{b} \qquad \text{Eq. 3.12}$$

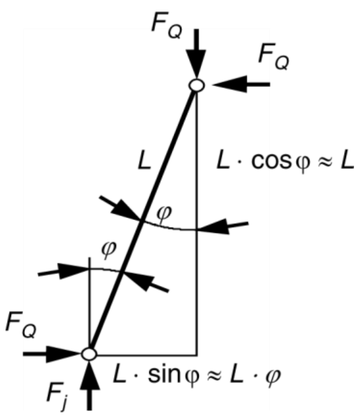

Fig. 3.7

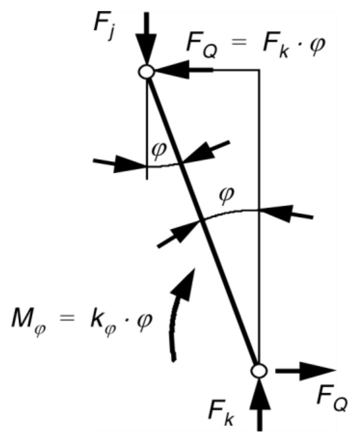

Fig. 3.8

The moment equilibrium about the joint applies for the free cut of the upper bar part.

$$\sum M_a = 0 = M_\varphi - F_j.L.\varphi = k_\varphi.\varphi - F_j.L.\varphi \qquad \text{Eq. 3.13}$$

At stability, this equilibrium condition must apply for non-zero displacement φ .

Equilibrium – upper bar

$$F_j.L.\varphi = F_Q.L \qquad \text{Eq. 3.14}$$

$$F_Q = F_j.\varphi \qquad \text{Eq. 3.15}$$

Equilibrium – lower bar

$$F_j.L.\varphi + F_j.L.\varphi = k_\varphi.\varphi \qquad \text{Eq. 3.16}$$

$$2.F_j.L = k_\varphi \qquad \text{Eq. 3.17}$$

$$F_j = \frac{k_\varphi}{2.L} \qquad \text{Eq. 3.18}$$

The buckling force results from Fj = Fk and Formula (3.12) inserted in Formula (3.18)

$$F_k = \frac{3.E.I}{2.\mathrm{b}.L} \qquad \text{Eq. 3.19}$$

8. Experimental Procedure

a. Experiment 1: Buckling with elastic joint

I. Experiment Aim

Determining the buckling force in the case of an elastic joint.

II. Preparing the Experiment

1. Clamp the upper bearing (9) on the lift vertical side of the frame. The height above the lower side of the frame is 600mm.
2. Clamp the lower joint bearing (8) straight below the hole of compression piece on the lower frame side.
3. Clamp the loading lever (5) with joint bearing (8) above the upper bearing (at 680mm height). The distance between the two bearings must be now 40mm.
4. Mount the compression piece (9a) in the upper bearing loosely. Insert the compression piece from the side with the clamping screws (9b)

5. Insert bar (7) between the lower bearing (8) and the upper compression piece (9a). The clip with the hook must point towards the lift side of the frame. The leaf spring (13) and the elbow (12) must be removed in advance.
6. Set the loading lever (5) horizontally and the gap between clamping screw (9b) and loading lever to 7mm.
7. Mount the sliding rider (6) on the loading lever.

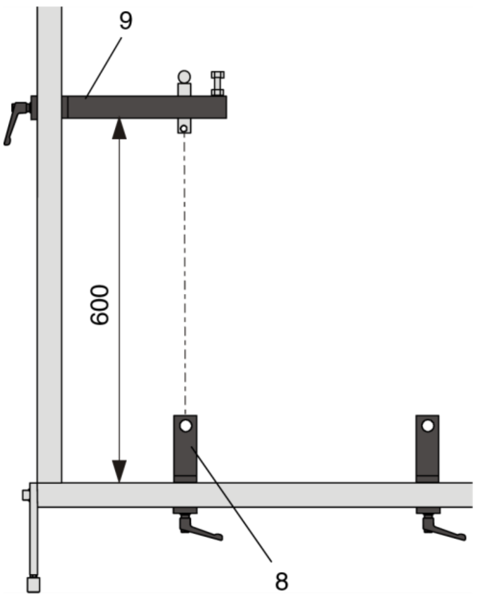

Fig. 3.9

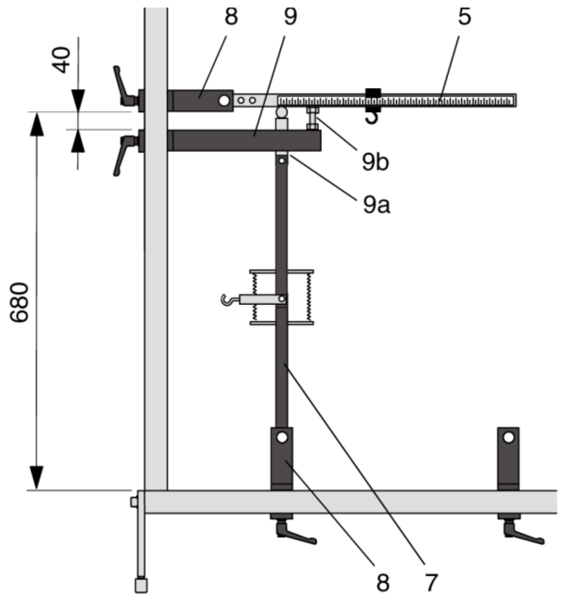

Fig. 3.10

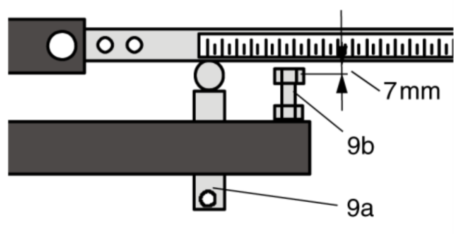

Fig. 3.11

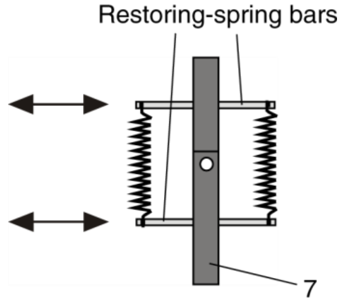

Fig. 3.12

First, a privileged direction of the buckling must be balanced.

1. For this purpose, load the buckling rod (7) with hand. The rod will buckle in a certain preferred direction.
2. In case of buckling towards lift, move the restoring-spring bars slightly to the lift.
3. In case of buckling towards right, move the restoring-spring bars slightly to the right.

4. Repeat the procedure until no significant preferred direction of buckling is noticed. The rod will slightly buckle in both directions alike.

Move the restoring-spring bars up and down equally.

III. Performing the Experiment

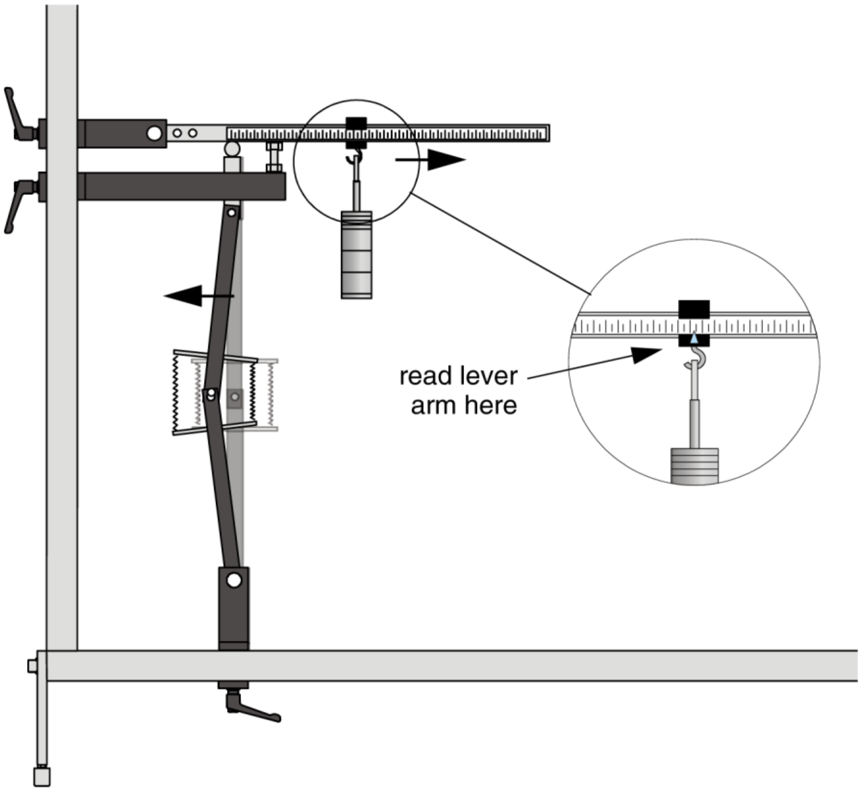

Fig. 3.13

1. Move the sliding rider the far lift (minimal lever arm).
2. Load the weight hanger with $3 \times 5N$ (total weight$16N$) and hang it in.
3. Increase the lever arm carefully by moving the sliding rider with the weight to the right.
4. Once the bar buckles read the lever arm and note it down.

b. Experiment 2: Buckling with Elastic Restraint

I. Experiment Aim

Determining the buckling force in the case of an elastic restraint.

II. Preparing the Experiment

Set the experiment in the same way as described in experiment 1.

1. Remove the springs on the center joint.
2. Fasten the bending spring (13) with the elbow (12) on the lower end of the buckling rod (7).
3. Move the joint bearing with the clamping device (14) from the right on the free leaf spring end.
4. Set the separation b between the two joint bearings to 200mm.

First, a privileged direction of the buckling must be balanced.

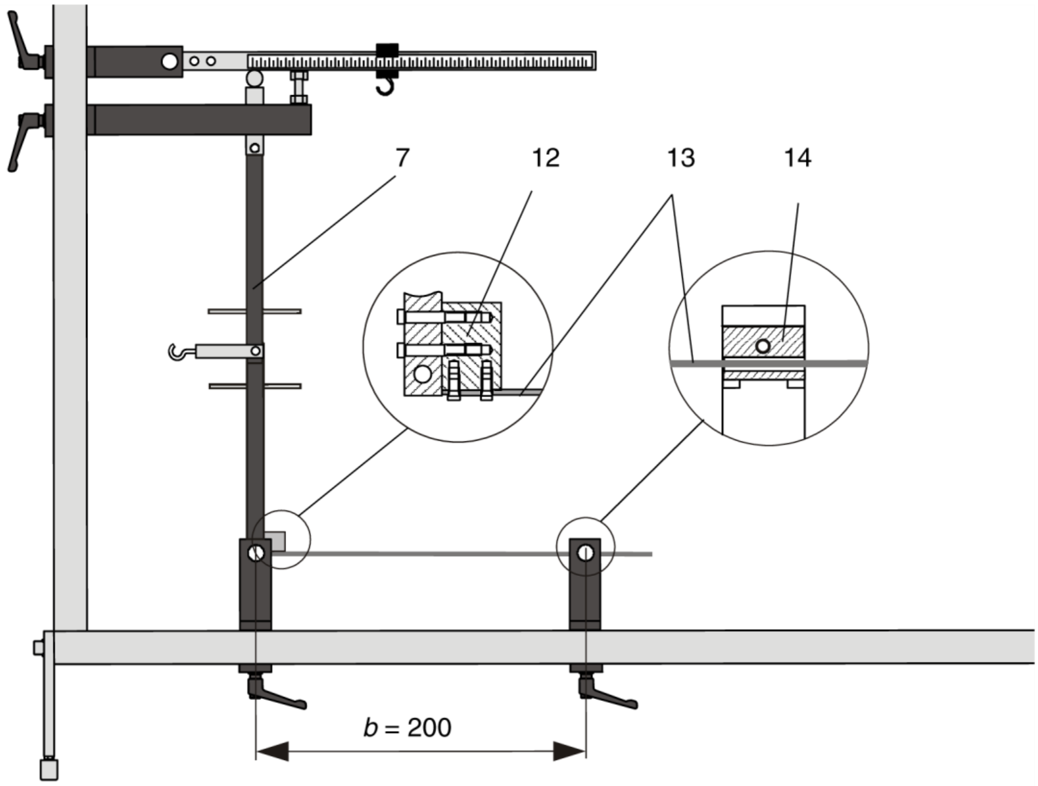

Fig. 3.14

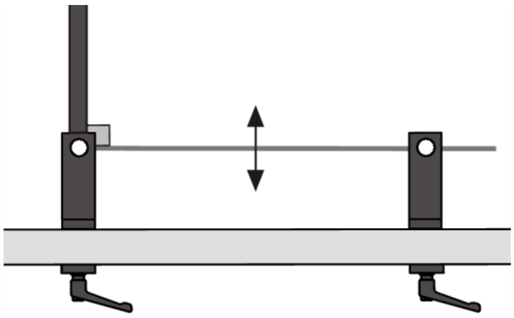

Fig. 3.15

1. For this purpose, load the buckling rod with hand. The rod will buckle in a certain preferred direction.

2. In case of buckling towards left, bend the leaf spring carefully down.
3. In case of buckling towards right, bend the leaf-spring carefully in upright direction.
4. Repeat the procedure until no significant preferred direction of buckling is noticed. The rod will slightly buckle in both directions alike.

III. Performing the Experiment

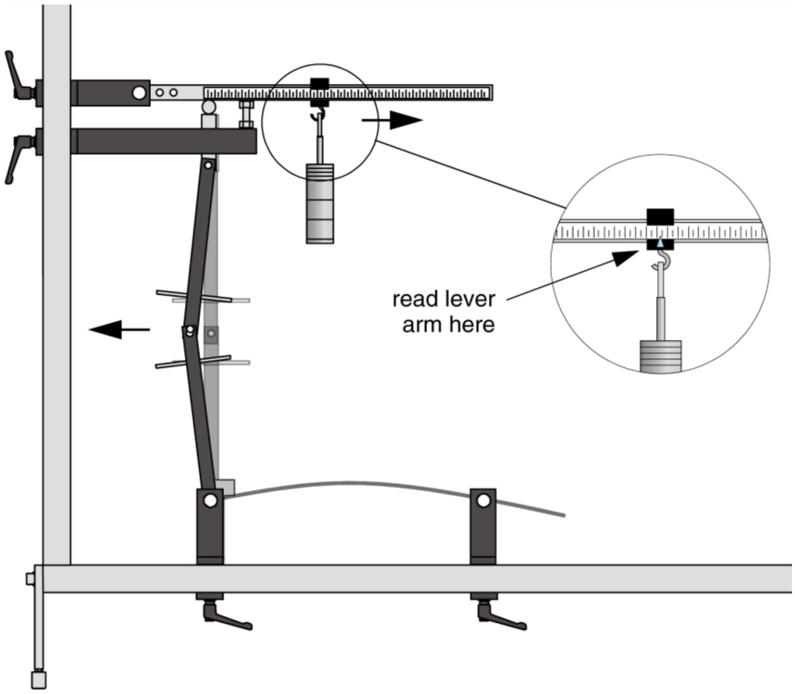

Fig. 3.16

1. Move the sliding rider the far lift (minimal lever arm).
2. Loa d the weight hanger with 2 x 5N (total weight 11N) and hang it in.
3. Increase the lever arm carefully by moving the sliding rider with the weight to the right.
4. Once the bar buckles read the lever arm and note it down.

c. Additional Experiments

The previous experiment can be repeated with various restraint stiffness. The stiffness can be adjusted by means of varying the separation between the two joint bearings. The remaining length of the leaf spring, i.e. the bearings separation must not be less than 150 mm.

A combination of elastic bearing and elastic restraint is also of interest.

One more experiment is the investigation of the influence of lateral forces on the stability. In this case, the pulley (11) is mounted. The tension rope (10) is fastened on the on the hook at the center joint.

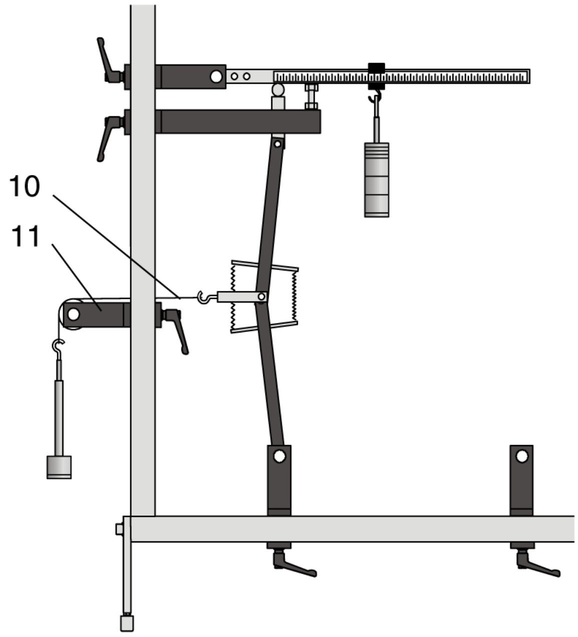

Fig. 3.17

9. Questions

1- Do the experiment 1 to determine the lever arm of the bar buckling then calculate the buckling force and compare it with calculate (Eq. 3.10).
2- Do the experiment 2 to determine the lever arm of the bar buckling then calculate the buckling force and compare it with calculate (Eq. 3.19).

10. Results

a. Experiment 1: Buckling with elastic joint

1- Mathematical calculation

2- Results

3- Discussion the results and conclusion:

b. Experiment 2: Buckling with Elastic Restraint

1- Mathematical calculation

2- Results

3- Discussion the results and conclusion: