Mechanics of Materials

Lecture 13

Combined Loadings

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Chapter Objectives

Analyze the stress developed in thin-walled pressure vessels

Review the stress analysis developed in previous chapters regarding axial load, torsion, bending and shear

Discuss the solution of problems where several of these internal loads occur simultaneously on a member's x-section

Chapter Outline

- ✓ Thin-Walled Pressure Vessels
- ✓ State of Stress Caused by Combined Loadings

Thin-Walled Pressure Vessels

Cylindrical or spherical vessels are commonly used in industry to serve as boilers or tanks. When under pressure, the material of which they are made is subjected to a loading from all directions. Although this is the case, the vessel can be analyzed in a simple manner provided it has a thin wall. In general, "***thin wall***" refers to a vessel having an inner-radius to wall-thickness ratio of 10 or more

Cylindrical vessel

Spherical vessel

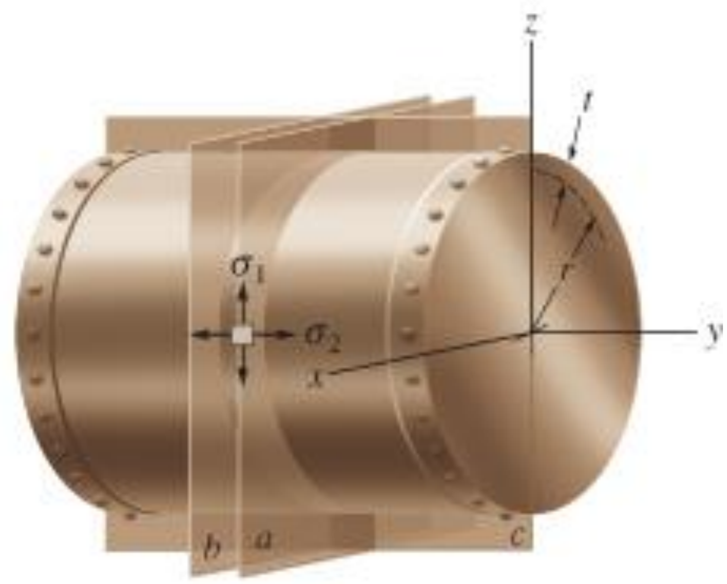
Thin-walled Pressure Vessels

- Assumption taken before analysis is that the thickness of the pressure vessel is uniform or constant throughout
- The pressure in the vessel is understood to be the gauge pressure, since it measures the pressure above atmospheric pressure, which is assumed to exist both inside and outside the vessel's wall

Thin-walled Pressure Vessels

Cylindrical vessels

- A gauge pressure p is developed within the vessel by a contained gas or fluid, and assumed to have negligible weight
- Due to uniformity of loading, an element of the vessel is subjected to normal stresses σ_1 in the *circumferential or hoop direction* and σ_2 in the *longitudinal or axial direction*

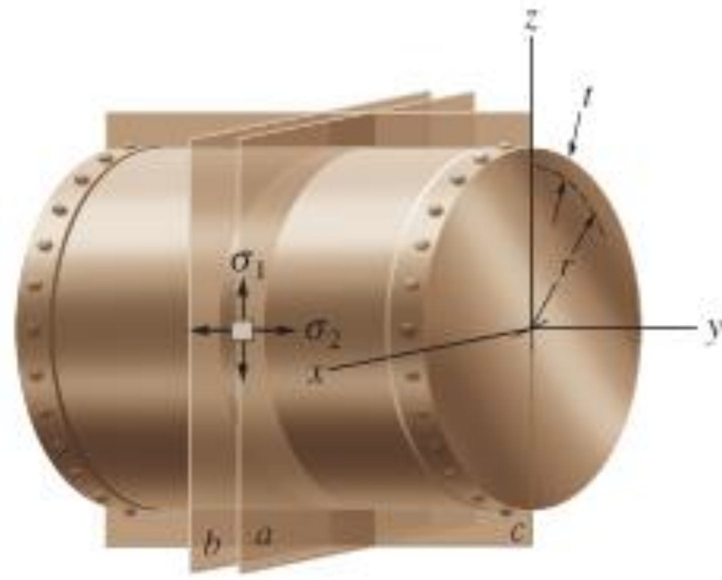


(a)

Thin-walled Pressure Vessels

Cylindrical vessels

- We use the method of sections and apply the equations of force equilibrium to get the magnitudes of the stress components.



(a)

$$\sigma_1 = p.r/t$$

$$\sigma_2 = p.r/2t$$

Thin-walled Pressure Vessels

Cylindrical vessels

$$\sigma_1 = p.r/t \qquad \sigma_2 = p.r/2t$$

σ_1, σ_2 = the normal stress in the hoop and longitudinal directions, respectively. Each is assumed to be *constant* throughout the wall of the cylinder, and each subjects the material to tension.

p = the internal gauge pressure developed by the contained gas

r = the inner radius of the cylinder

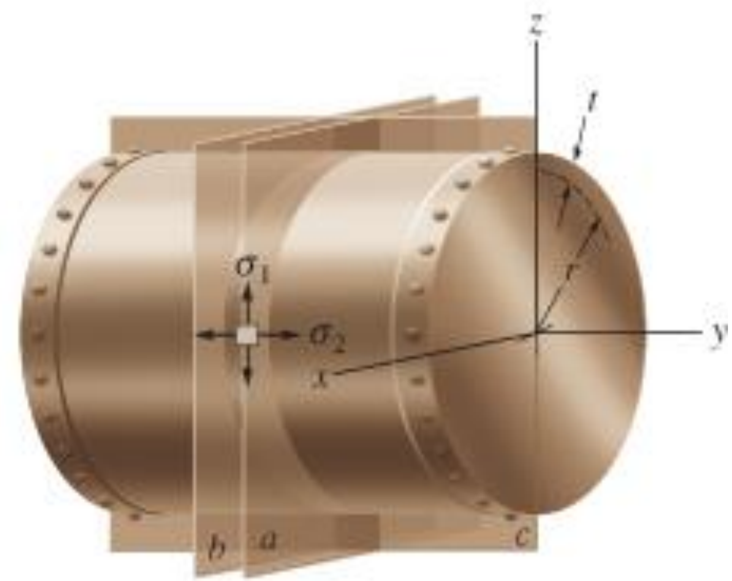
t = the thickness of the wall

(a)

Thin-walled Pressure Vessels

Cylindrical vessels

- The hoop or circumferential stress is twice as large as the longitudinal or axial stress
- When engineers fabricate cylindrical pressure vessels from rolled-form plates, the longitudinal joints must be designed to carry twice as much stress as the circumferential joints



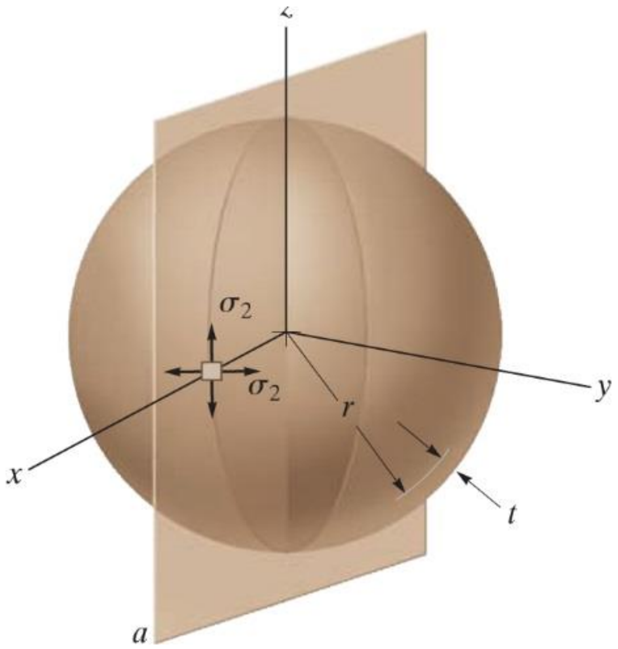
(a)

Thin-walled Pressure Vessels

Spherical Vessels

- The analysis for a spherical pressure vessel can be done in a similar manner
- Like the cylinder, equilibrium in the y direction requires

$$\sigma_2 = p.r/2t$$



Spherical Vessels

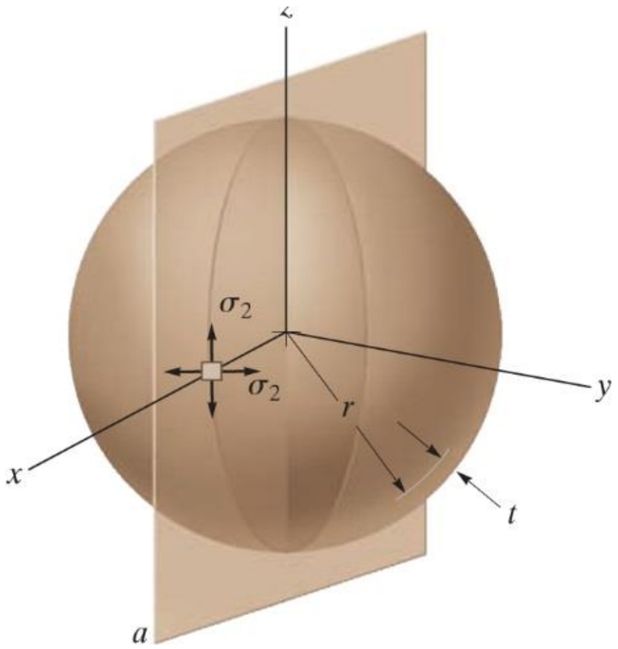
$$\sigma_2 = p.r/2t$$

σ_2 = the normal stress in the hoop and longitudinal directions, it is assumed to be *constant* throughout the wall of the Sphere, and it subjects the material to tension.

p = the internal gauge pressure developed by the contained gas

r = the inner radius of the cylinder

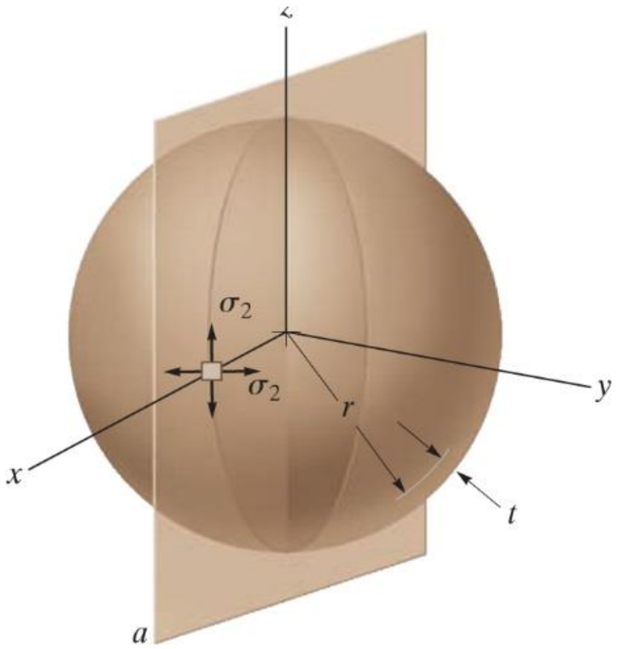
t = the thickness of the wall



Thin-walled Pressure Vessels

Spherical Vessels

- This stress is the same regardless of the orientation of the hemispheric free-body diagram.
- An element of material taken from either a cylindrical or spherical pressure vessel is subjected to biaxial stress; normal stress existing in two directions



Thin-walled Pressure Vessels

- The material is also subjected to radial stress, $\sigma 3$. It has a value equal to pressure p at the interior wall and decreases to zero at exterior surface of the vessel.
- However, we ignore the radial stress component for thin-walled vessels, since the limiting assumption of $r/t = 10$, results in $\sigma 1$ and $\sigma 2$ being 5 and 10 times higher than maximum radial stress $(\sigma 3)max = p$

State of Stress Caused by Combined Loadings

Most often, the cross section of a member is subjected to several of simple loadings simultaneously. When this occurs, the method of superposition can be used to determine the resultant stress distribution. Remember that the principle of superposition can be used for this purpose provided a linear relationship exists between the stress and the loads.

Also, the geometry of the member should not undergo significant change when the loads are applied. These conditions are necessary in order to ensure that the stress produced by one load is not related to the stress produced by any other load.

State of Stress Caused by Combined Loadings

Application of the method of superposition

- The stress distribution due to each loading is determined
- These distributions are superimposed to determine the resultant stress distribution

Conditions to satisfy

- A *linear relationship* exists between the stress and the loads
- Geometry of the member should *not* undergo *significant change* when the loads are applied

State of Stress Caused by Combined Loadings

- This is necessary to ensure that the stress produced by one load is not related to the stress produced by any other load

State of Stress Caused by Combined Loadings

Procedure for Analysis

The following procedure provides a general means for establishing the normal and shear stress components at a point in a member when the member is subjected to several different types of loadings simultaneously. It is assumed that the material is homogeneous and behaves in a linear elastic manner. Also, Saint-Venant's principle requires that the point where the stress is to be determined is far removed from any discontinuities in the cross section or points of applied load.

Procedure for Analysis

Internal Loading.

- Section the member perpendicular to its axis at the point where the stress is to be determined and obtain the resultant internal normal and shear force components and the bending and torsional moment components.
- The force components should act through the centroid of the cross section, and the moment components should be computed about centroidal axes, which represent the principal axes of inertia for the cross section.

Procedure for Analysis

Stress Components.

- Determine the stress component associated with each internal loading. For each case, represent the effect either as a distribution of stress acting over the entire cross-sectional area, or show the stress on an element of the material located at a specified point on the cross section.

State of Stress Caused by Combined Loadings

Normal Force.

- The internal normal force is developed by a uniform normal-stress distribution determined from

$$\sigma = P/A$$

Shear Force.

- The internal shear force in a member is developed by a shear-stress distribution determined from the shear formula,

$$\tau = V.Q/I.t$$

Special care, however, must be exercised when applying this equation,

State of Stress Caused by Combined Loadings

Bending Moment.

- For straight members the internal bending moment is developed by a normal-stress distribution that varies linearly from zero at the neutral axis to a maximum at the outer boundary of the member. This stress distribution is determined from the flexure formula,

$$\sigma = M.y/I$$

State of Stress Caused by Combined Loadings

Torsional Moment.

- For circular shafts and tubes the internal torsional moment is developed by a shear-stress distribution that varies linearly from the central axis of the shaft to a maximum at the shaft's outer boundary. This stress distribution is determined from the torsional formula,

$$\tau = T.\rho / J_P$$

State of Stress Caused by Combined Loadings

Thin-Walled Pressure Vessels.

- If the vessel is a thin-walled cylinder, the internal pressure p will cause a biaxial state of stress in the material such that the hoop or circumferential stress component is $\sigma_1 = p.r/t$ and the longitudinal stress component is $\sigma_2 = p.r/2t$. If the vessel is a thin- walled sphere, then the biaxial state of stress is represented by two equivalent components, each having a magnitude of

$$\sigma_2 = p.r/2t$$

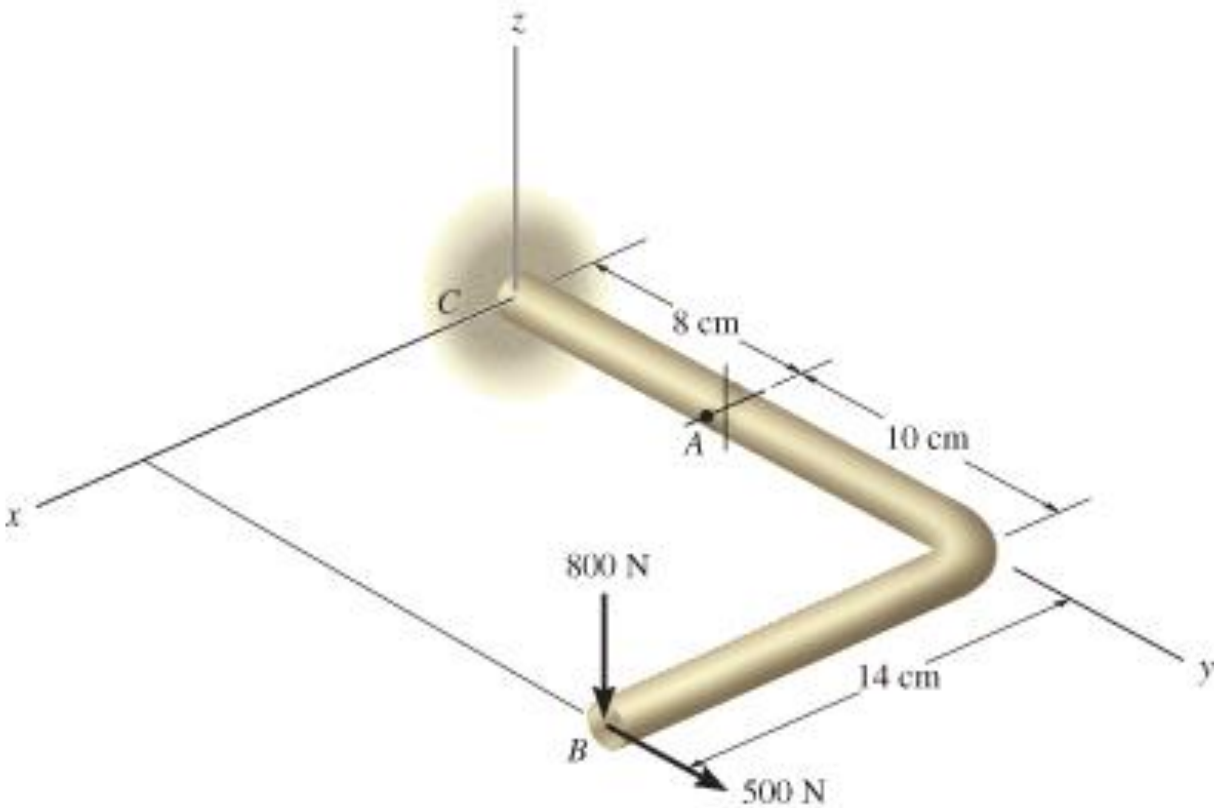
State of Stress Caused by Combined Loadings

Superposition.

- Once the normal and shear stress components for each loading have been calculated, use the principle of superposition and determine the resultant normal and shear stress components.
- Represent the results on an element of material located at the point, or show the results as a distribution of stress acting over the member's cross-sectional area.

Example

The solid rod shown has a radius of 0.75 cm. If it is subjected to the loading shown, determine the stress at point *A*.

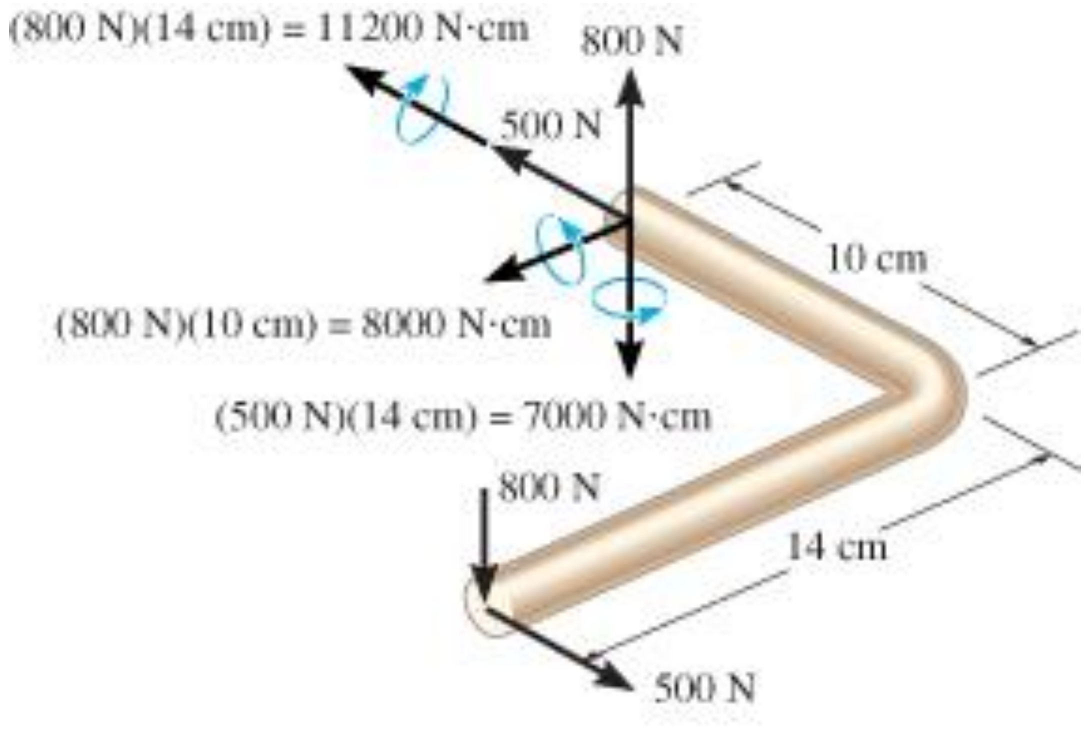


(a)

Example

Internal loadings

Rod is sectioned through point *A*. Using free-body diagram of segment *AB*, the resultant internal loadings can be determined from the six equations of equilibrium.



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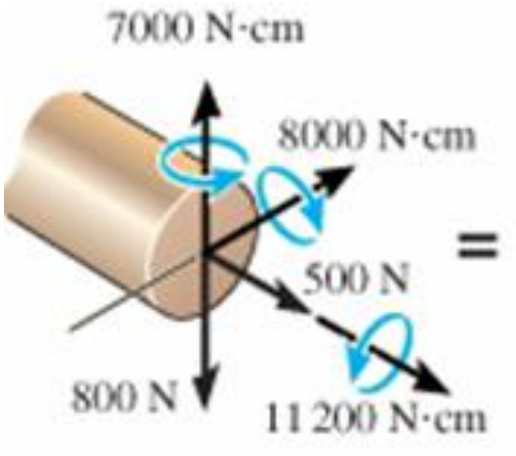
Example

Internal loadings

The normal force (500 N) and shear force (800 N) must act through the centroid of the x-section and the bending-moment components (8000 N·cm) and 7000 N·cm) are applied about centroidal (principal) axes. In order to better "visualize" the stress distributions due to each of these loadings, we will consider the *equal but opposite resultants* acting on *AC*.



Combined loading

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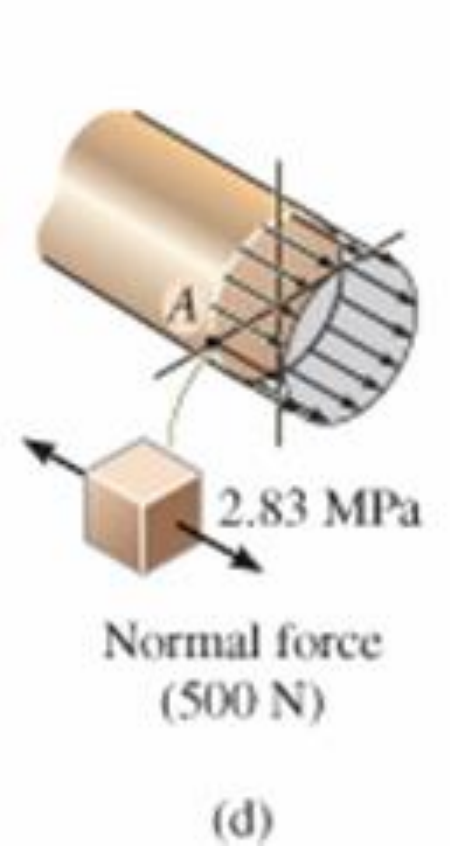
Example

Stress components

1. Normal force

Normal stress distribution is shown.
For point A, we have

$$\sigma_A = P/A = \ldots = 2.83 \text{ MPa}$$



Example

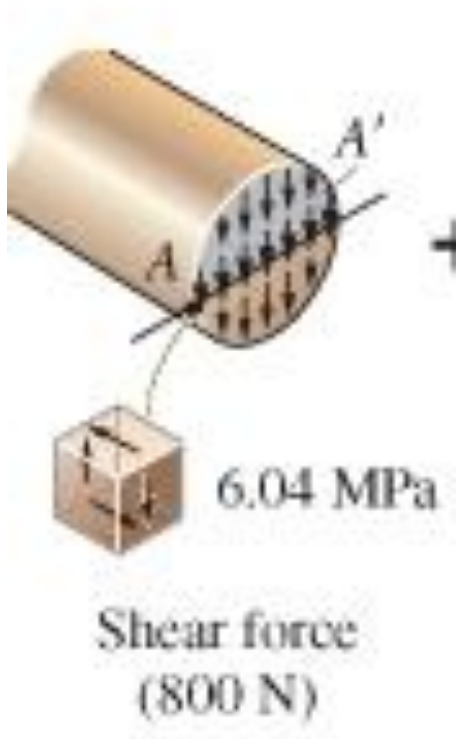
Stress components

2. Shear force

Shear-stress distribution is shown. For point *A*, *Q* is determined from the shaded semicircular area. Using tables provided in textbook, we have

$$Q = y'\overline{A}' = \ldots = 0.2813\ \text{cm}^3$$

$$\tau_A = VQ/It = \ldots = 6.04\ \text{MPa}$$



(e)

Example

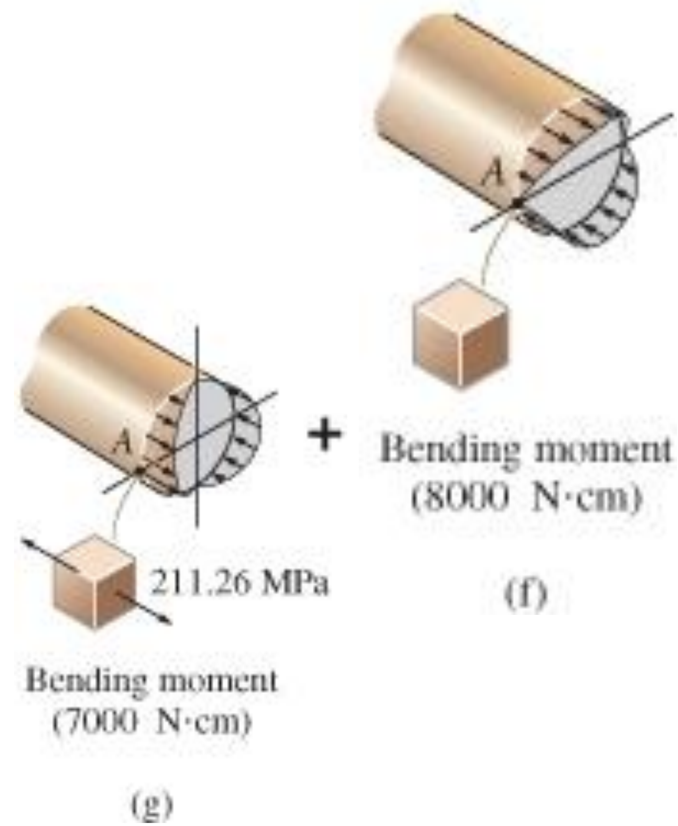
Stress components

3. Bending moments

For the 8000 N·cm component, point *A* lies on the neutral axis, so the normal stress is $\sigma_A = 0$

For the 7000 N·cm component, $c = 0.75$ cm, so normal stress at point *A*, is

$$\sigma_A = Mc/I = \ldots = 211.26 \text{ MPa}$$



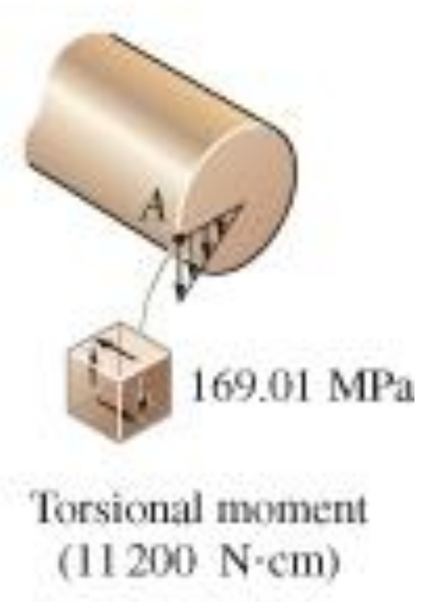
Example

Stress components

4. Torsional moment

At point *A*, $\rho_A = c = 0.75$ cm, thus

$$\tau_A = Tc/J = \ldots = 169.01 \text{ MPa}$$



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Example

Superposition

When the above results are superimposed, it is seen that an element of material at point *A* is subjected to both normal and shear stress components