Mechanics of Materials

Lecture 12

Transverse Shear

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Chapter Objectives

- ✓ Develop a method for finding the shear stress in a beam having a prismatic x-section and made from homogeneous material that behaves in a linear-elastic manner
- ✓ This method of analysis is limited to special cases of x-sectional geometry

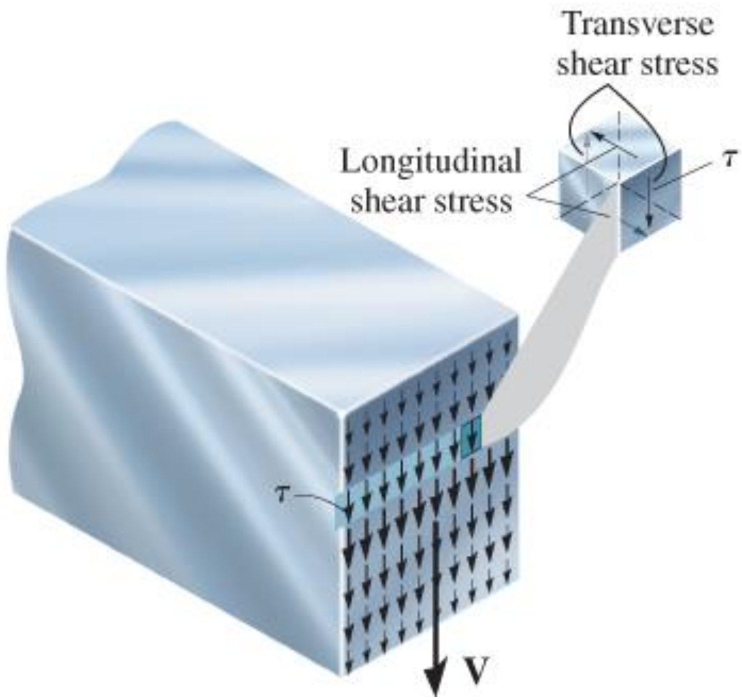
- ✓ Discuss the concept of shear flow, with shear stress for beams and thin-walled members
- ✓ Discuss the shear center

Chapter Outline

- ✓ Shear in Straight Members
- ✓ The Shear Formula
- ✓ Shear Stresses in Beams
- ✓ Shear Flow in Built-up Members
- ✓ Shear Flow in Thin-Walled Members
- ✓ Shear Center

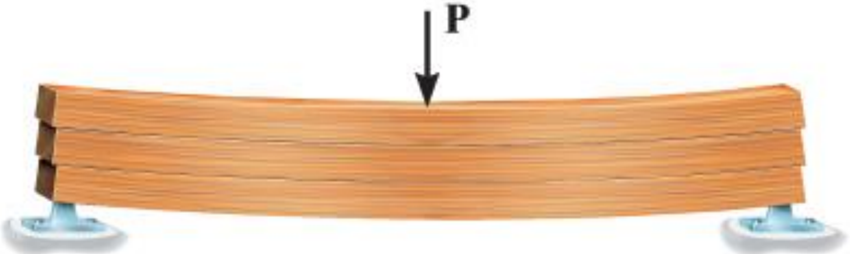
Shear in Straight Members

- Shear **V** is the result of a transverse shear-stress distribution that acts over the beam's x-section.
- Due to complementary property of shear, associated longitudinal shear stresses also act along longitudinal planes of beam

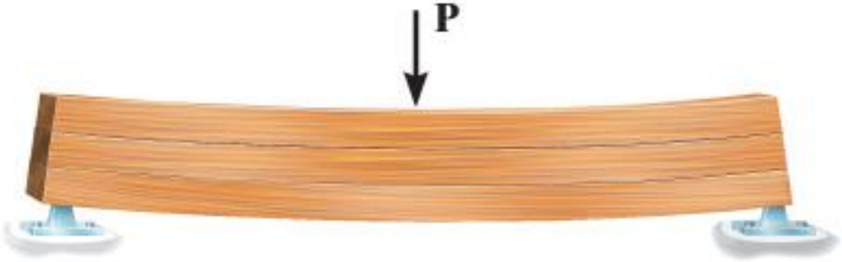


Shear in Straight Members

- As shown below, if top and bottom surfaces of each board are smooth and not bonded together, then application of load **P** will cause the boards to slide relative to one another.
- However, if boards are bonded together, longitudinal shear stresses will develop and distort x-section in a complex manner



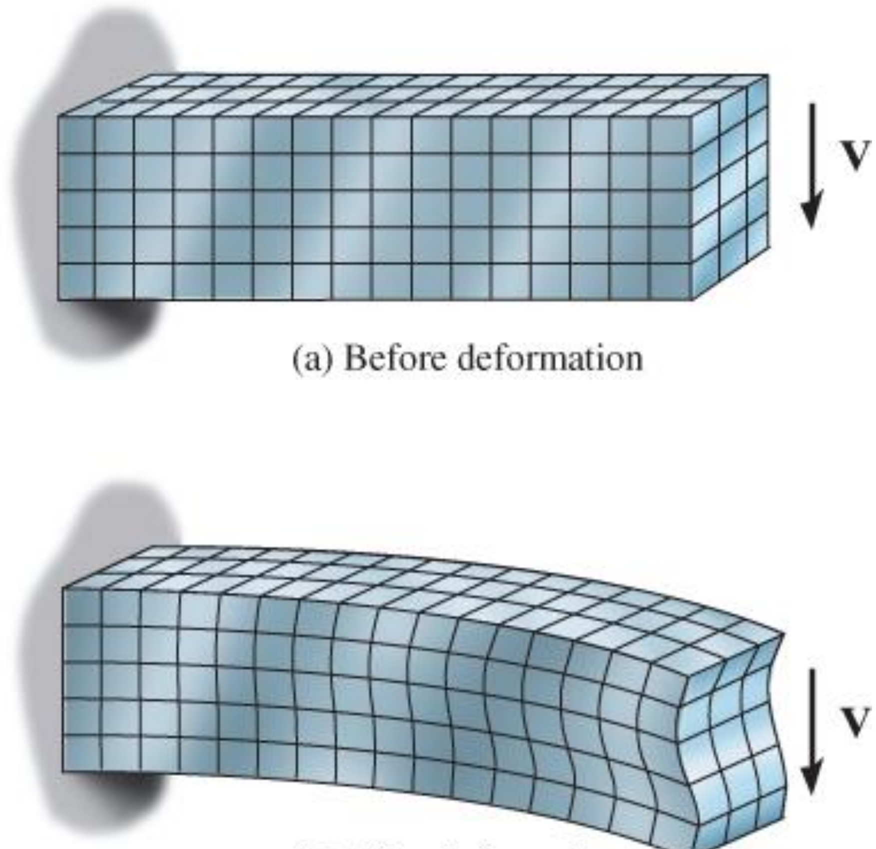
Boards not bonded together
(a)



Boards bonded together
(b)

Shear in Straight Members

As shown, when shear **V** is applied, the non-uniform shear-strain distribution over x-section will cause it to warp, i.e., not remain in plane.

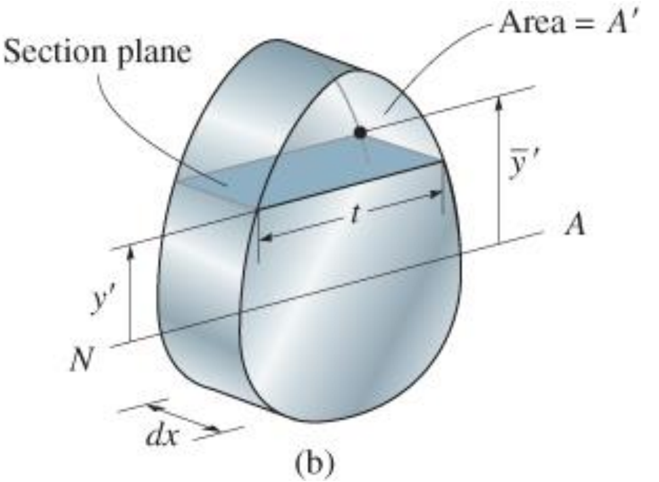


Shear in Straight Members

- Recall that the flexure formula assumes that x-sections must remain plane and perpendicular to longitudinal axis of beam after deformation
- This is violated when beam is subjected to both bending and shear, we assume that the warping is so small it can be neglected. This is true for a slender beam (small depth compared with its length)
- For transverse shear, shear-strain distribution throughout the depth of a beam cannot be easily expressed mathematically

The Shear Formula

$$\tau = \frac{VQ}{It}$$



$\tau =$ shear stress in member at the point located a distance y' from the neutral axis. Assumed to be constant and therefore *averaged* across the width t of member

$V =$ internal resultant shear force, determined from method of sections and equations of equilibrium

$I =$ moment of inertia of entire x-sectional area computed about the neutral axis

$t =$ width of the member's x-sectional area, measured at the point where τ is to be determined

$Q = \int_{A'} y\, dA' = \bar{y}'A'$, where A' is the top (or bottom) portion of member's x-sectional area, defined from section where t is measured, and $\bar{y}'$ is distance of centroid of A', measured from neutral axis

The Shear Formula

- The equation derived is called the *shear formula*
- Since the equation is derived indirectly from the flexure formula, the material must behave in a linear-elastic manner and have a modulus of elasticity that is the *same* in tension and in compression
- Shear stress in composite members can also be obtained using the shear formula
- To do so, compute Q and I from the *transformed section* of the member. Thickness t in formula remains the actual width t of x-section at the point where τ is to be calculated

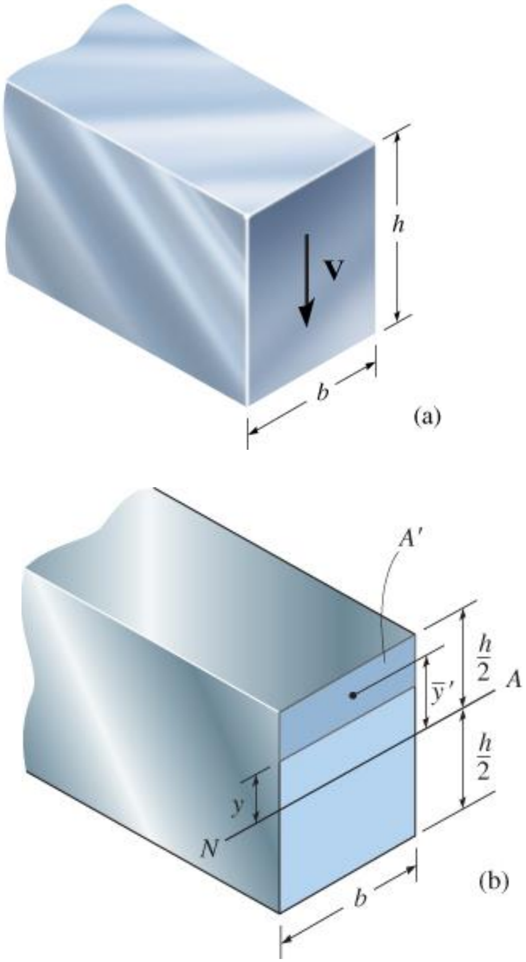
Shear Stresses in Beams

Rectangular x-section

Consider beam to have rectangular x-section of width *b* and height *h* as shown.

Distribution of shear stress throughout x-section can be determined by computing shear stress at arbitrary height *y* from neutral axis, and plotting the function. Hence,

$$Q = \frac{1}{2}\left(\frac{h^2}{4} - y^2\right)b$$



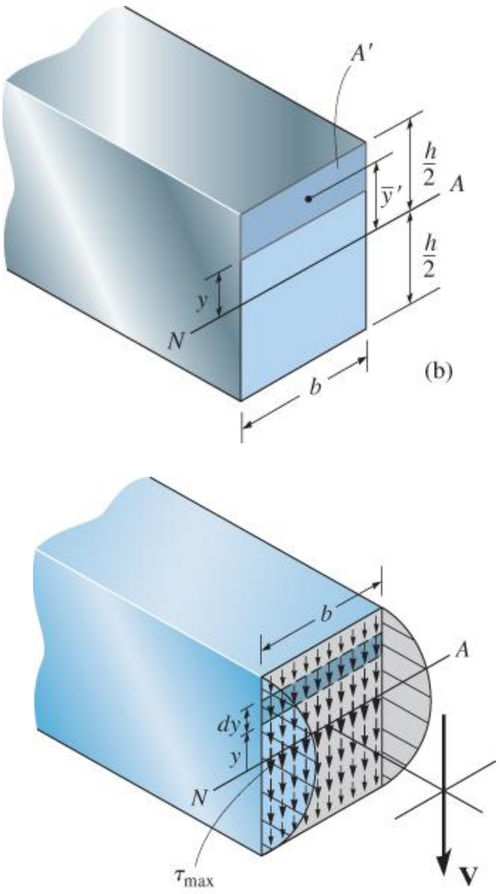
Shear Stresses in Beams

Rectangular x-section

- After deriving Q and applying the shear formula, we have

$$\left.\begin{aligned} Q &= \frac{1}{2}\left(\frac{h^2}{4} - y^2\right)b \\ \tau &= \frac{VQ}{It} \\ I &= \frac{bh^3}{12} \end{aligned}\right\} \tau = \frac{6V}{bh^3}\left(\frac{h^2}{4} - y^2\right)$$

- This equation indicates that shear-stress distribution over x-section is parabolic.



Shear–stress distribution

(c)

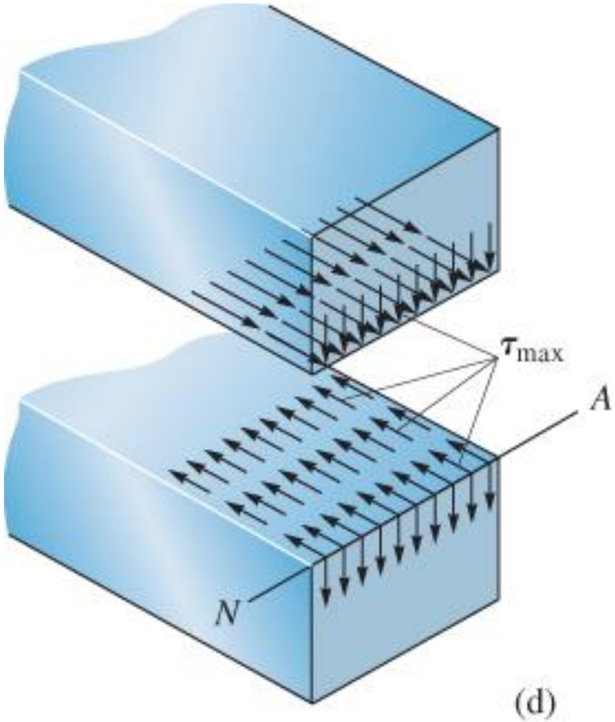
Shear Stresses in Beams

Rectangular x-section

- At $y = 0$, we have

$$\tau = \frac{6V}{bh^3}\left(\frac{h^2}{4}\right) = 1.5\frac{V}{bh}$$

- By comparison, τ_{max} is 50% greater than the average shear stress determined from τ_{avg} = V/A.

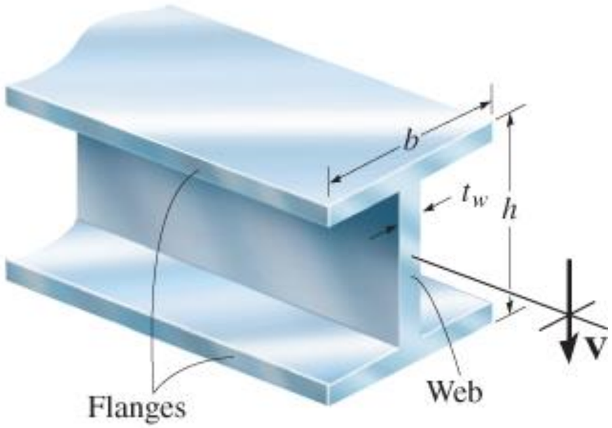


(d)

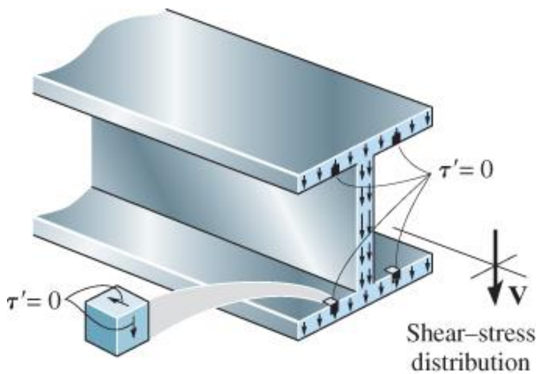
Shear Stresses in Beams

Wide-flange beam

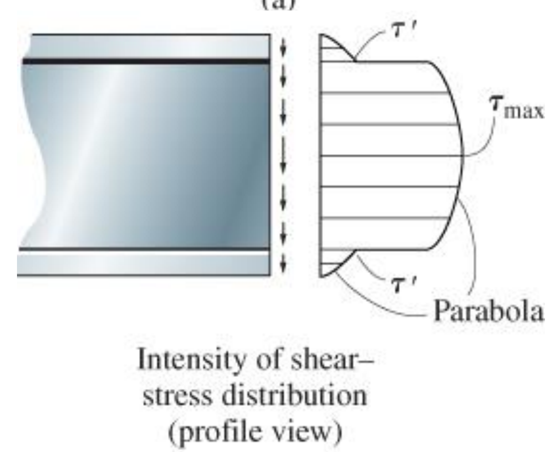
- A wide-flange beam consists of two (wide) “flanges” and a “web”.
- Using analysis similar to a rectangular x-section, the shear stress distribution acting over x-section is shown



(a)



(b)

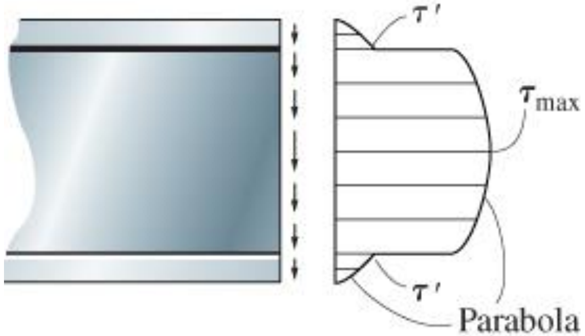


(c)

Shear Stresses in Beams

Wide-flange beam

- The shear-stress distribution also varies parabolically over beam's depth
- Note there is a jump in shear stress at the flange-web junction since x-sectional thickness changes at this point
- The web carries significantly more shear force than the flanges



Intensity of shear–stress distribution (profile view)

(c)

Shear Stresses in Beams

Limitations on use of shear formula

- One major assumption in the development of the shear formula is that shear stress is uniformly distributed over width t at section where shear stress is to be determined
- By comparison with exact mathematical analysis based on theory of elasticity, the magnitude difference can reach 40%
- This is especially so for the flange of a wide-flange beam

Shear Stresses in Beams

Limitations on use of shear formula

- The shear formula will also give inaccurate results for the shear stress at the flange-web junction of a wide-flange beam, since this is a point of sudden x-sectional change (stress concentration occurs here)
- Furthermore, inner regions of flanges are free boundaries, thus shear stress at these boundaries should be zero
- However, shear formula calculated at these points will not be zero

Shear Stresses in Beams

Limitations on use of shear formula

- Fortunately, engineers are often interested in the average maximum shear stress, which occurs at the neutral axis, where *b/h* ratio is very small
- Also, shear formula does not give accurate results when applied to members having x-sections that are short or flat, or at points where the x-section suddenly changes
- It should also not be applied across a section that intersects the boundary of a member at an angle other than 90^o

Shear Stresses in Beams

IMPORTANT

- Shear forces in beams cause non-linear shear-strain distributions over the x-section, causing it to warp
- Due to complementary property of shear stress, the shear stress developed in a beam acts on both the x-section and on longitudinal planes
- The shear formula was derived by considering horizontal force equilibrium of longitudinal shear stress and bending-stress distributions acting on a portion of a differential segment of the beam

Shear Stresses in Beams

IMPORTANT

- The shear formula is to be used on straight prismatic members made of homogeneous material that has linear-elastic behavior. Also, internal resultant shear force must be directed along an axis of symmetry for x-sectional area
- For beam having rectangular x-section, shear stress varies parabolically with depth.
- For beam having rectangular x-section, maximum shear stress is along neutral axis

Shear Stresses in Beams

IMPORTANT

- Shear formula should not be used to determine shear stress on x-sections that are short or flat, or at points of sudden x-sectional changes, or at a point on an inclined boundary

Procedure for analysis

Internal shear

- Section member perpendicular to its axis at the point where shear stress is to be determined
- Obtain internal shear **V** at the section

Section properties

- Determine location of neutral axis, and determine the moment of inertia I of entire x-sectional area about the neutral axis
- Pass an imaginary horizontal section through the point where the shear stress is to be determined

Procedure for analysis

Section properties

- Measure the width t of the area at this section
- Portion of area lying either above or below this section is A'.
- Determine Q either by integration, $Q = \int_{A'} y \, dA'$, or by using $Q = \bar{y}'A'$.
- Here, $\bar{y}'$ is the distance of centroid of A', measured from the neutral axis. (TIP: A' is the portion of the member's x-sectional area being "held onto the member" by the longitudinal shear stresses.)

Procedure for analysis

Shear stress

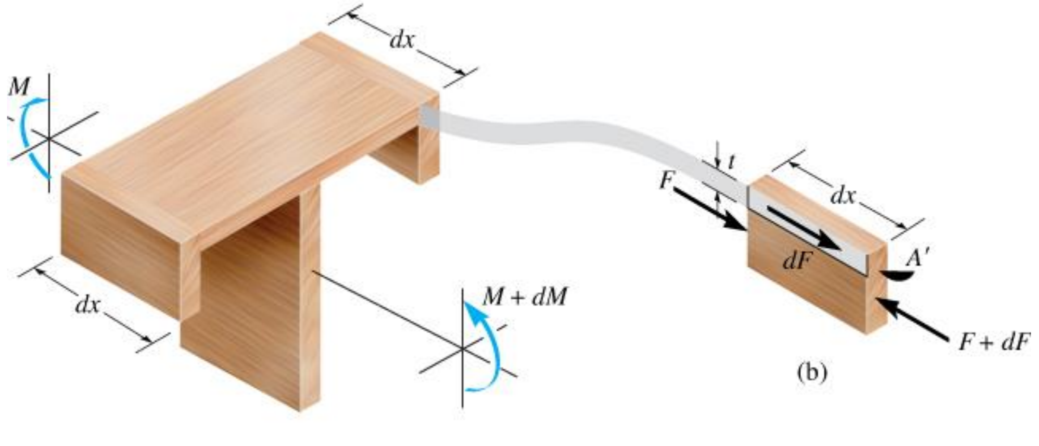
- Using consistent set of units, substitute data into the shear formula and compute shear stress τ
- Suggest that proper direction of transverse shear stress be established on a volume element of material located at the point where it is computed
- τ acts on the x-section in the same direction as **V**. From this, corresponding shear stresses acting on the three other planes of element can be established

Shear Flow in Built-up Members

- Occasionally, in engineering practice, members are "built-up" from several composite parts in order to achieve a greater resistance to loads, some examples are shown.
- If loads cause members to bend, fasteners may be needed to keep component parts from sliding relative to one another.
- To design the fasteners, we need to know the shear force resisted by fastener along member's *length*

Shear Flow in Built-up Members

- This loading, measured as a force per unit length, is referred to as the *shear flow q*.
- Magnitude of shear flow along any longitudinal section of a beam can be obtained using similar development method for finding the shear stress in the beam



(a)

Shear Flow in Built-up Members

- Thus shear flow is

$$q = VQ/I$$

q = shear flow, measured as a force per unit length along the beam

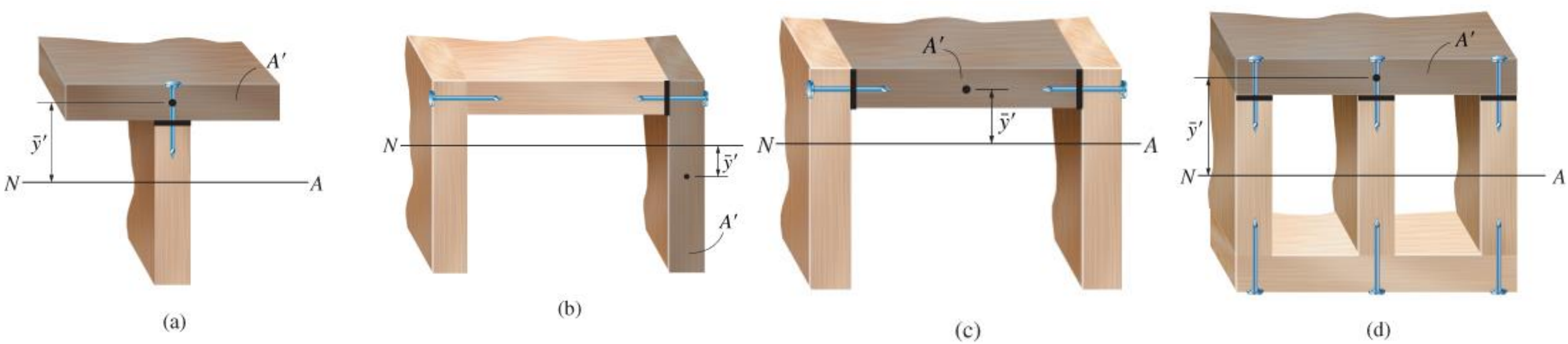
V = internal resultant shear force, determined from method of sections and equations of equilibrium

I = moment of inertia of entire x-sectional area computed about the neutral axis

$Q = \int_{A'} y\, dA' = y'A'$, where A' is the x-sectional area of segment connected to beam at juncture where shear flow is to be calculated, and y' is distance from neutral axis to centroid of A'

Shear Flow in Built-up Members

- Note that the fasteners in (a) and (b) supports the calculated value of q
- And in (c) each fastener supports $q/2$
- In (d) each fastener supports $q/3$



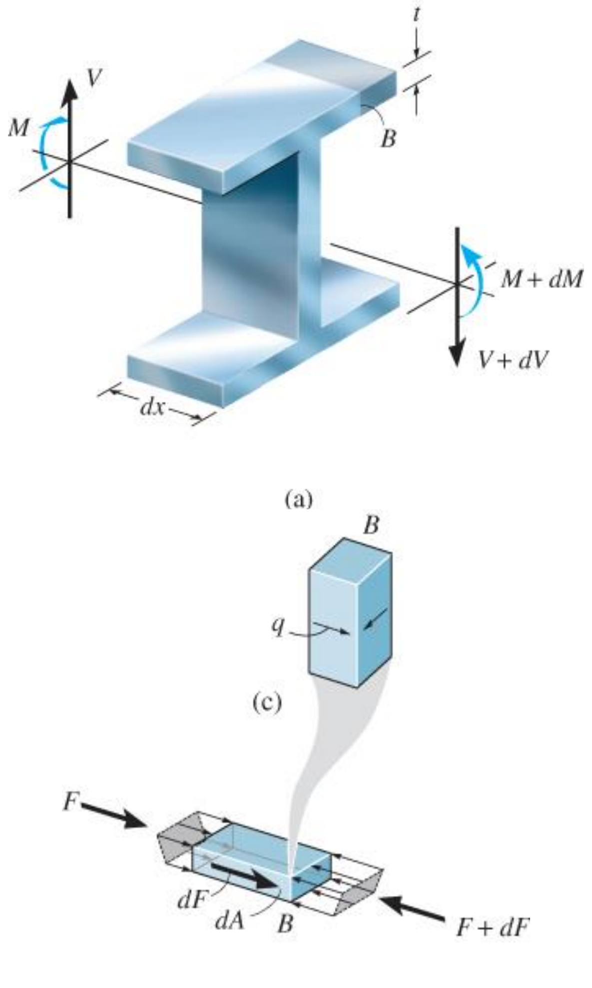
Shear Flow in Built-up Members

IMPORTANT

- Shear flow is a measure of force per unit length along a longitudinal axis of a beam.
- This value is found from the shear formula and is used to determine the shear force developed in fasteners and glue that holds the various segments of a beam together

Shear Flow in Thin-walled Members

- We can use shear-flow equation $q = VQ/I$ to find the shear-flow distribution throughout a member's x-sectional area.
- We assume that the member has thin walls, i.e., wall thickness is small compared with height or width of member



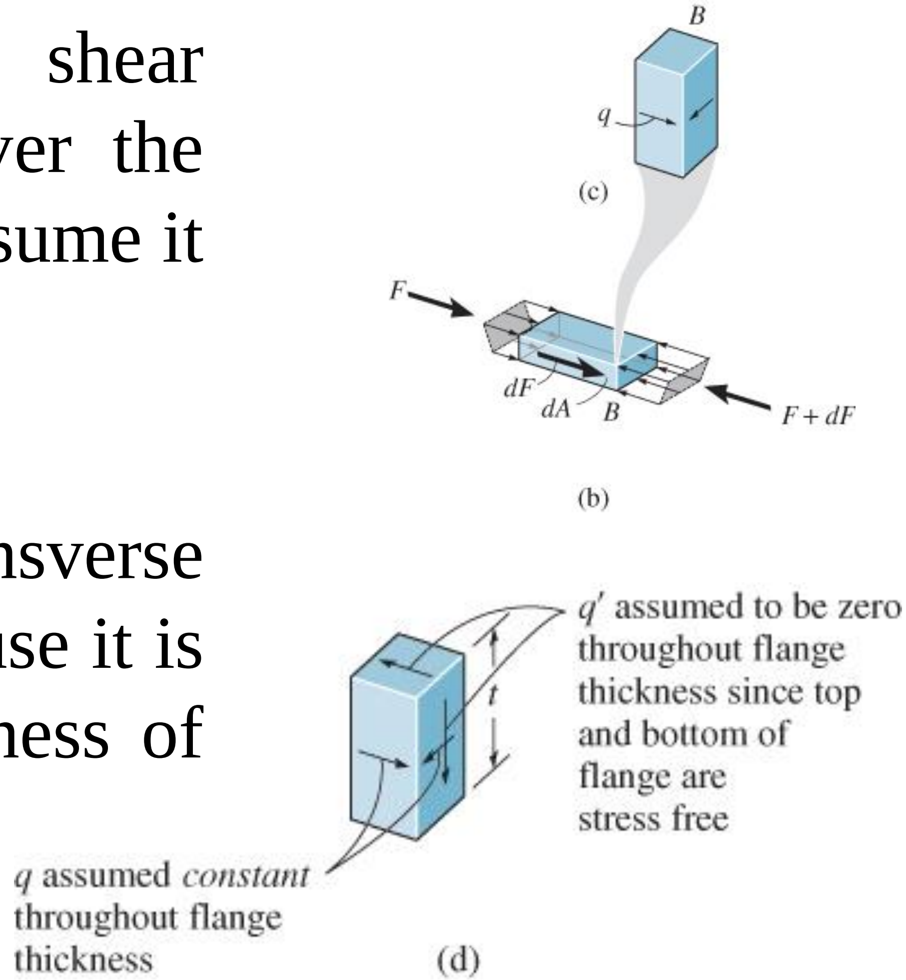
(b)

Shear Flow in Thin-walled Members

Because flange wall is thin, shear stress will not vary much over the thickness of section, and we assume it to be constant. Hence,

$$q = \tau \times t$$

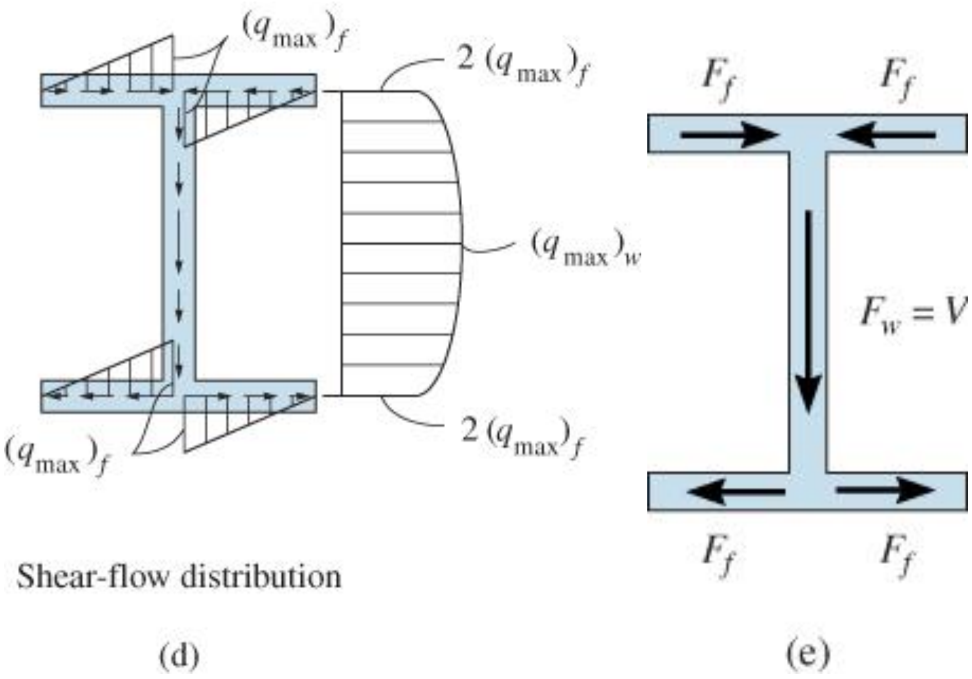
We will neglect the vertical transverse component of shear flow because it is approx. zero throughout thickness of element



Shear Flow in Thin-walled Members

- To determine distribution of shear flow along top right flange of beam, shear flow is

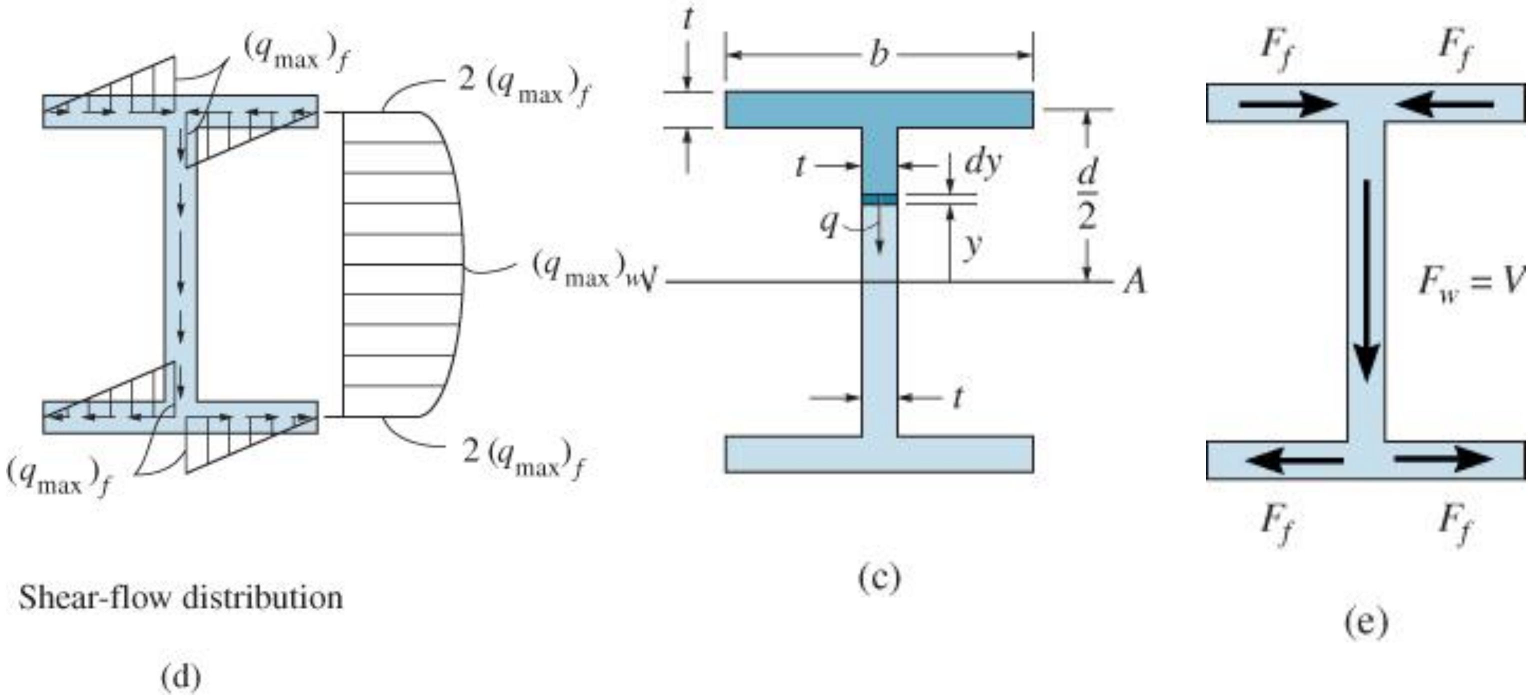
$$q = \frac{Vtd}{2I}\left(b/2 - x\right)$$



Shear Flow in Thin-walled Members

- Similarly, for the web of the beam, shear flow is

$$q = \frac{V\,t}{I}\left[d\,b/2 + 0.5\left(d^2/4 - y^2\right)\right]$$

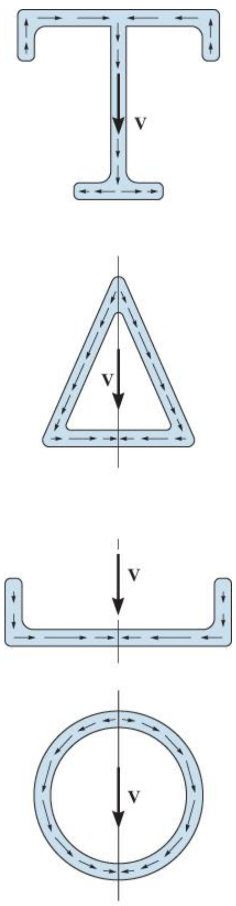


Shear Flow in Thin-walled Members

- Value of q changes over the x-section, since Q will be different for each area segment A'
- q will vary linearly along segments (flanges) that are perpendicular to direction of **V**, and parabolically along segments (web) that are inclined or parallel to **V**
- q will always act parallel to the walls of the member, since section on which q is calculated is taken perpendicular to the walls

Shear Flow in Thin-walled Members

- Directional sense of *q* is such that shear appears to "flow" through the x-section, inward at beam's top flange, "combining" and then "flowing" downward through the web, and then separating and "flowing" outward at the bottom flange



Shear flow q

Shear Flow in Thin-walled Members

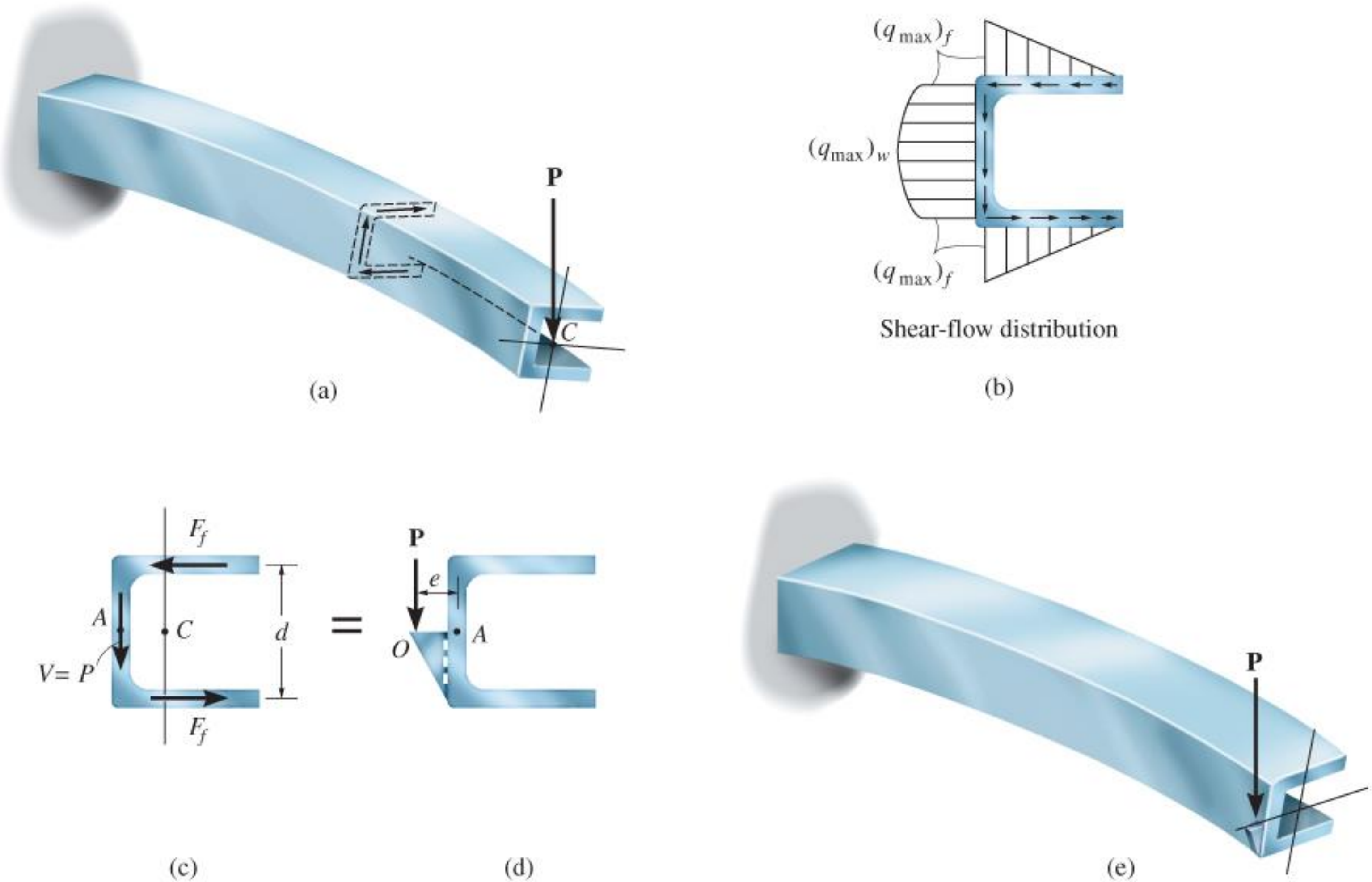
IMPORTANT

- If a member is made from segments having thin walls, only the shear flow parallel to the walls of member is important
- Shear flow varies linearly along segments that are perpendicular to direction of shear **V**
- Shear flow varies parabolically along segments that are inclined or parallel to direction of shear **V**
- On x-section, shear "flows" along segments so that it contributes to shear **V** yet satisfies horizontal and vertical force equilibrium

Shear Center

- Previously, we assumed that internal shear **V** was applied along a principal centroidal axis of inertia that *also* represents the *axis of symmetry* for the x-section
- Here, we investigate the effect of applying the shear along a principal centroidal axis that is not an axis of symmetry
- When a force P is applied to a channel section along the once vertical unsymmetrical axis that passes through the *centroid C* of the x-sectional area, the channel bends downwards and also *twist clockwise*

Shear Center



Shear Center

- When the shear-flow distribution is integrated over the flange and web areas, a resultant force of F_f in each flange and a force of $V=P$ in the web is created
- If we sum the moments of these forces about point A, the couple (or torque) created by the flange forces causes the member to twist
- To prevent the twisting, we need to apply **P** at a point O located a distance e from the web of the channel, thus

$$\sum M_A = F_f.d = P.e \Rightarrow e = \frac{F_f.d}{P}$$

Shear Center

- Express F_f is expressed in terms of **P** (= **V**) and dimensions of flanges and web to reduce *e* as a function of its x-sectional geometry
- We name the point *O* as the *shear center or flexural center*
- When **P** is applied at the shear center, beam will bend without twisting
- Note that *shear center will always lie on an axis of symmetry* of a member's x-sectional area

Shear Center

IMPORTANT

- Shear center is the point through which a force can be applied which will cause a beam to bend and yet not twist
- Shear center will always lie on an axis of symmetry of the x-section
- Location of the shear center is only a function of the geometry of the x-section and does not depend upon the applied loading

Procedure for analysis

Shear-flow resultants

- Magnitudes of force resultants that create a moment about point *A* must be calculated
- For each segment, determine the shear flow q at an arbitrary point on segment and then integrate q along the segment's length
- Note that **V** will create a linear variation of shear flow in segments that are perpendicular to **V** and a parabolic variation of shear flow in segments that are parallel or inclined to **V**

Procedure for analysis

Shear-flow resultants

- Determine the direction of shear flow through the various segments of the x-section
- Sketch the force resultants on each segment of the x-section
- Since shear center determined by taking the moments of these force resultants about a point (A), choose this point at a location that eliminates the moments of as many as force resultants as possible

Procedure for analysis

Shear center

- Sum the moments of the shear-flow resultants about point A and set this moment equal to moment of **V** about point A
- Solve this equation and determine the moment-arm distance e, which locates the line of action of **V** from point A
- If axis of symmetry for x-section exists, shear center lies at the point where this axis intersects line of action of **V**

Procedure for analysis

Shear center

- If no axes of symmetry exists, rotate the x-section by 90^o and repeat the process to obtain another line of action for **V**
- Shear center then lies at the point of intersection of the two 90^o lines