Mechanics of Materials

Lecture 8

Bending (1)

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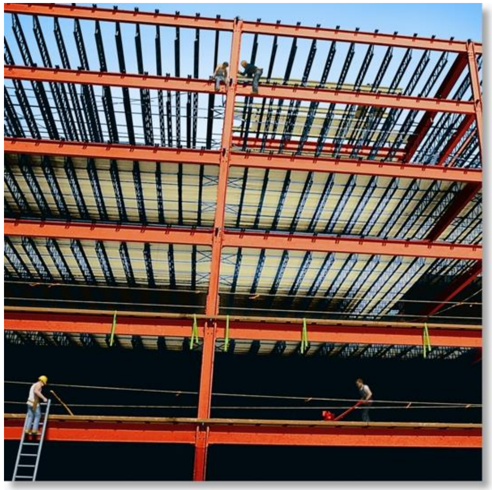
Lecture Objectives



- ✓ Determine stress in members caused by bending
- ✓ Discuss how to establish shear and moment diagrams for a beam or shaft
- ✓ Determine largest shear and moment in a member, and specify where they occur
- ✓ Consider members that are straight, symmetric x-section and homogeneous linear-elastic material
- ✓ Consider special cases of unsymmetrical bending and members made of composite materials

Lecture Outline

- ✓ Shear and Moment Diagrams
- ✓ Graphical Method for Constructing Shear and Moment Diagrams
- ✓ Bending Deformation of a Straight Member
- ✓ The Flexure Formula
- ✓ Unsymmetrical Bending

Loading Stats

- **Loading stats**
  - **Axial**

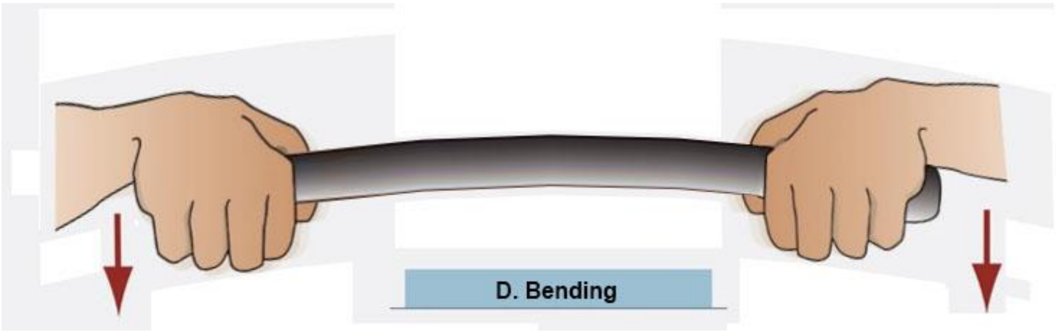
    The load applied along its longitudinal axis
    - **Tension**
    - **Compression**
    - **Buckling**
  - **Bending**

    The load applied perpendicular to its longitudinal axis
  - **Torsion**

    Torque is a moment that tends to twist a member about its longitudinal axis

Bending Loading

Bending



Beams

Beams : Members that are slender and support loadings that are applied perpendicular to their longitudinal axis; They are classified as to how they are supported.

a simply supported beam is pinned at one end and roller supported at the other,

Simply supported beam

a cantilevered beam is fixed at one end and free at the other,

Cantilevered beam

an overhanging beam has one or both of its ends freely extended over the supports.

Overhanging beam

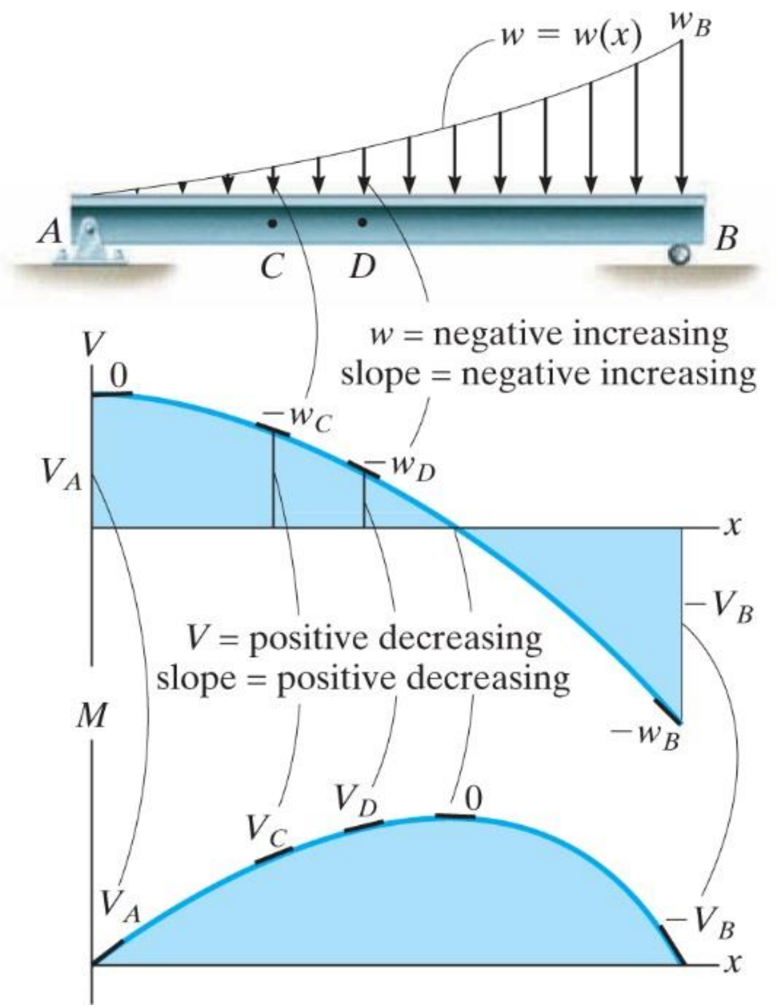
Beams

Beams are considered among the most important of all structural elements.

They are used to support the floor of a building, the deck of a bridge, or the wing of an aircraft. Also, the axle of an automobile, the boom of a crane, even many of the bones of the body act as beams.

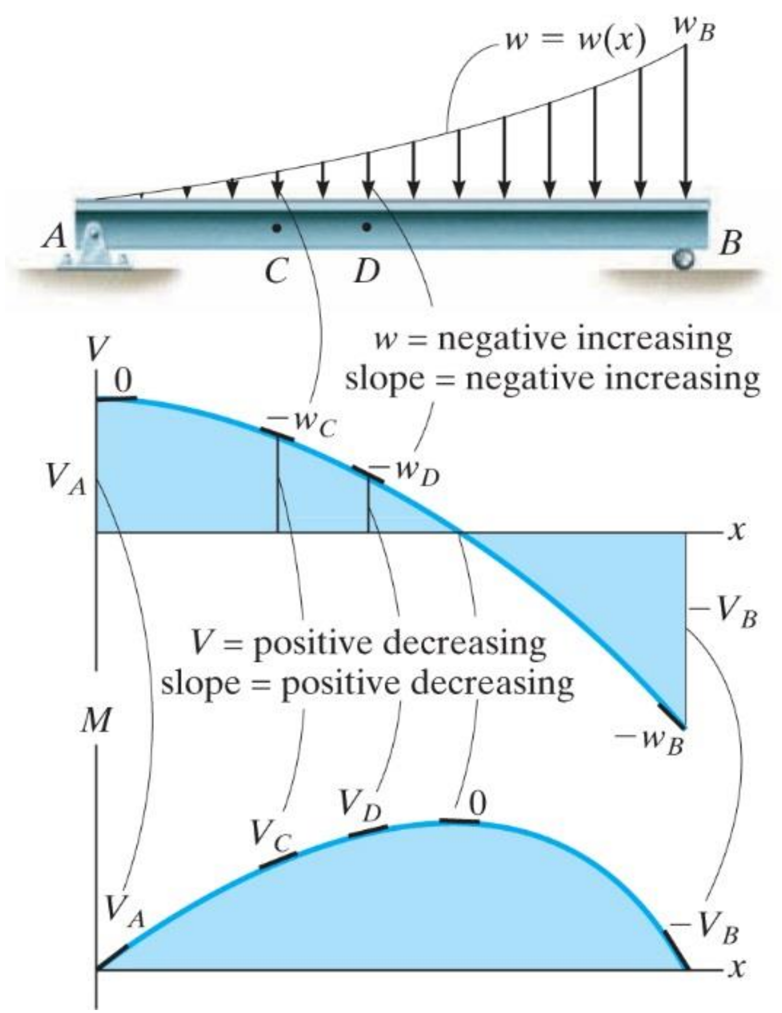
Beams

Because of the applied loadings, beams develop an internal **shear force** and **bending moment** that, in general, vary from point to point along the axis of the beam. In order to properly design a beam it therefore becomes necessary to determine the maximum shear and moment in the beam.



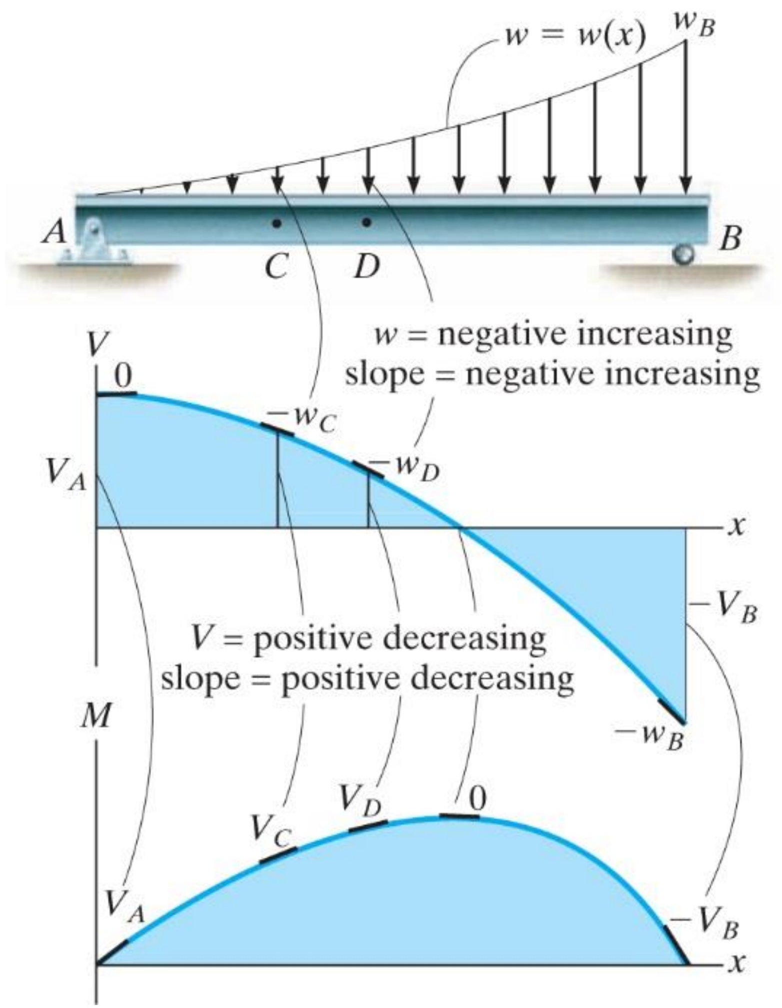
Beams

One way to do this is to express V and M as functions of their arbitrary position x along the beam's axis. These shear and moment functions can then be plotted and represented by graphs called shear and moment diagrams. The maximum values of V and M can then be obtained from these graphs.



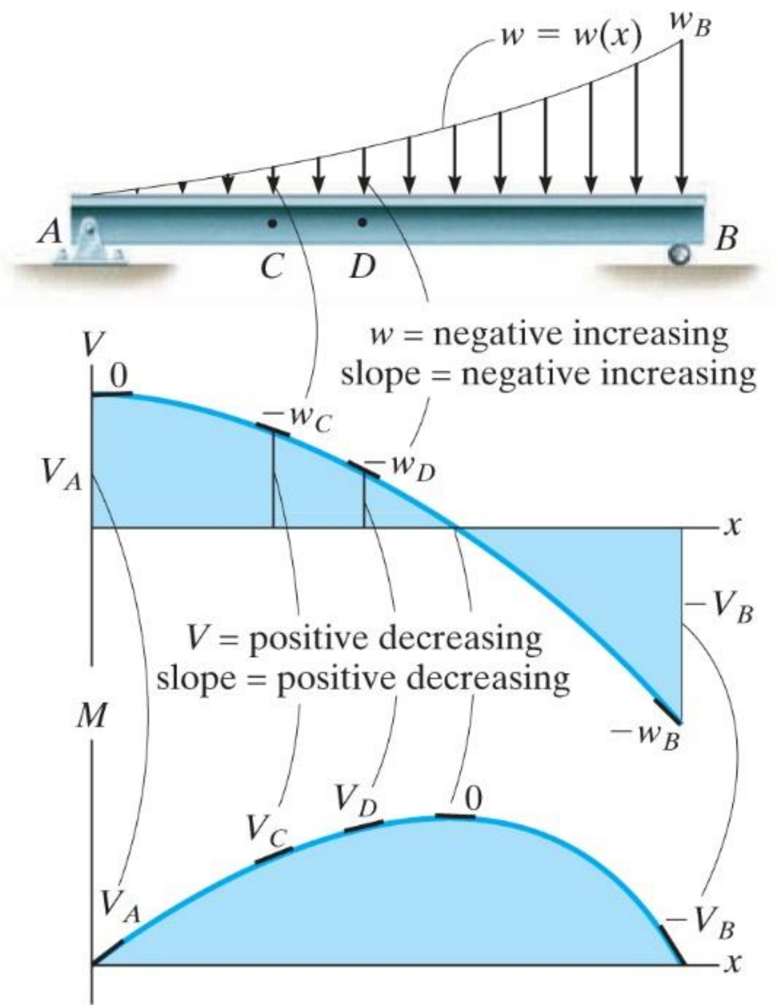
Beams

The shear and moment diagrams are often used by engineers to decide where to place reinforcement materials within the beam or how to proportion the size of the beam at various points along its length.

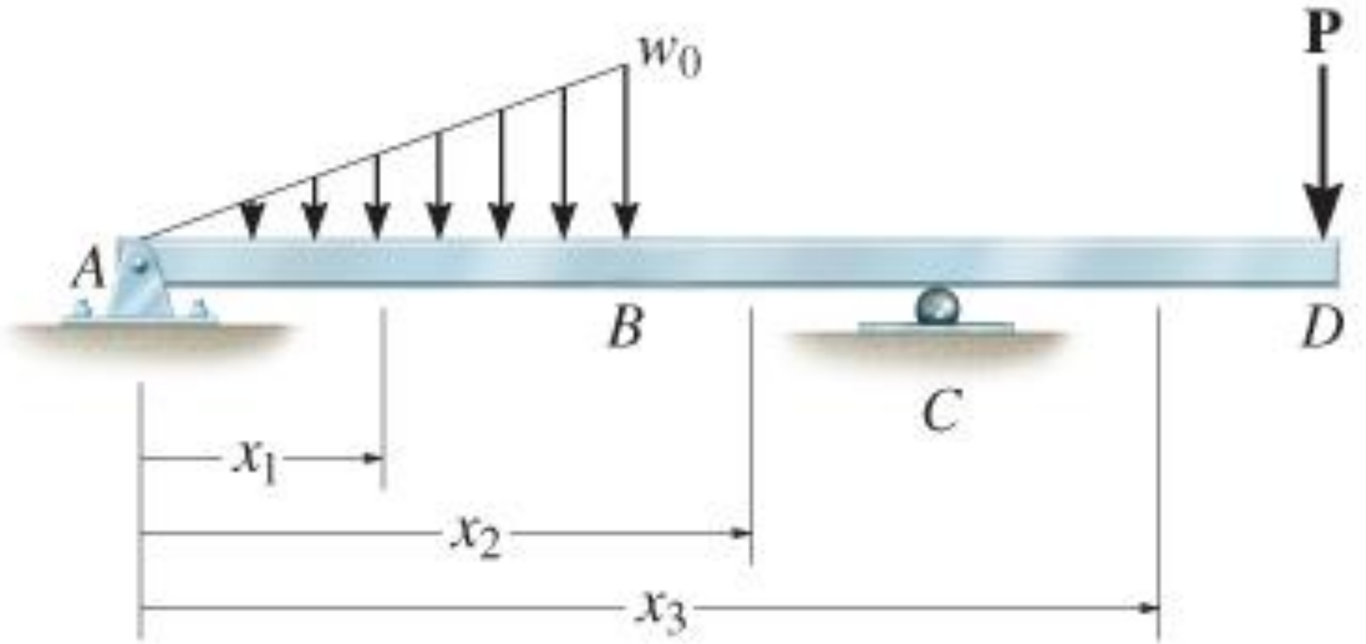


Beams

In general, the internal shear and moment functions of x will be discontinuous, at points where a distributed load changes or where concentrated forces or couple moments are applied.



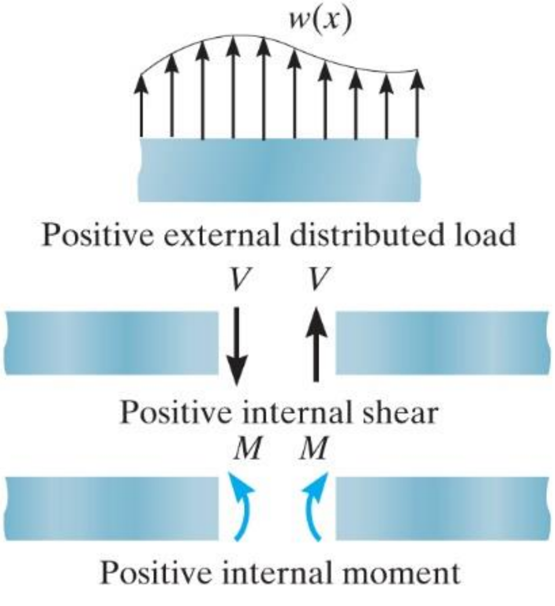
Shear and bending-moment functions must be determined for each region of the beam between any two discontinuities of loading.



Beam Sign Convention

The choice of a sign convention is arbitrary, the sign convention used in engineering practice consider that the positive directions are as follows:

- The distributed load acts upward on the beam;
- The internal shear force causes a clockwise rotation of the beam segment on which it acts;
- The internal moment causes compression in the top fibers of the segment

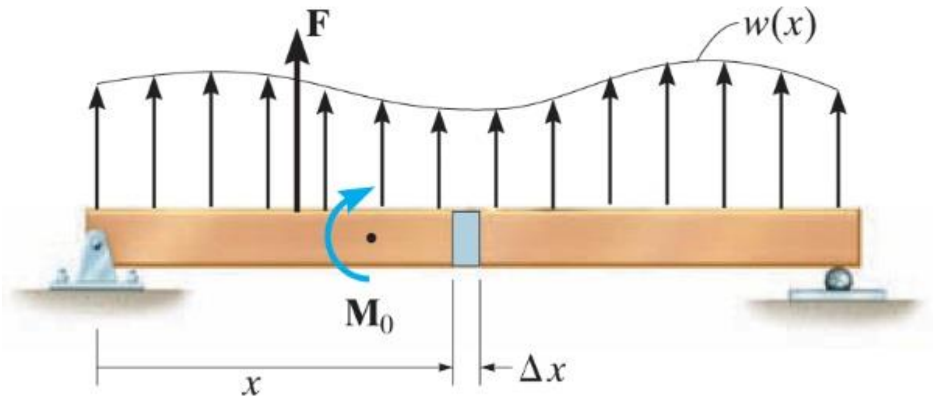


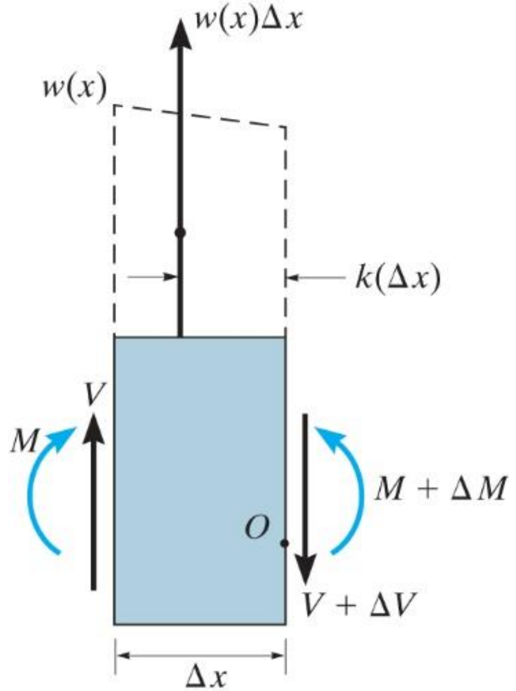
Beam sign convention

Loadings that are opposite to these are considered negative.

Shear and Moment Diagrams

Consider a beam subjected to an arbitrary loading.





Free-body diagram of segment Δx

A free-body diagram for a small segment Δx chosen at a position x where there is no concentrated force or couple moment. The distributed load has been replaced by a resultant force w(x).Δx that acts at a fractional distance k(Δx) from the right side.

Shear and Moment Diagrams

$$\sum F_y = 0;$$

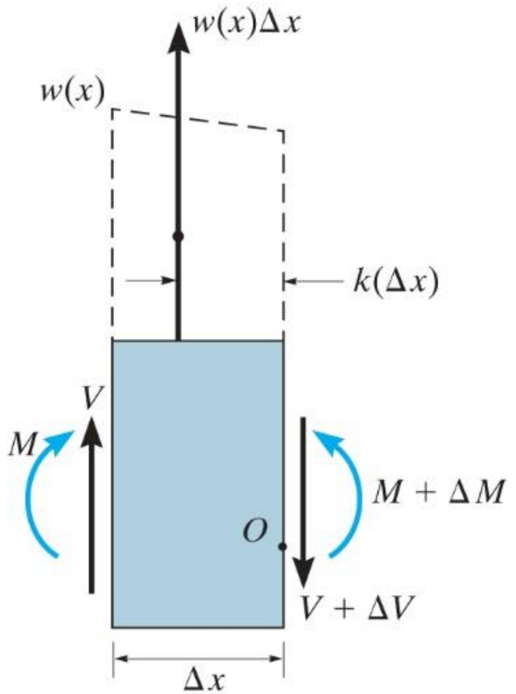
$$V + w(x).\Delta x - (V + \Delta V) = 0$$

$$\Delta V = w(x).\Delta x$$

$$\sum M_O = 0;$$

$$-V.\Delta x - M + w(x).\Delta x\left[k(\Delta x)\right] + (M + \Delta M) = 0$$

$$\Delta M = V.\Delta x$$



Free-body diagram of segment Δx

Dividing by Δx and taking the limit as $\Delta x \to 0$ the above two equations become

$$\frac{dM}{dx} = V \qquad \frac{dV}{dx} = w(x)$$

Shear and Moment Diagrams

$$\frac{dV}{dx} = w(x)$$

$$\frac{dM}{dx} = V$$

$$\begin{matrix}\text{slope of shear} \\ \text{diagram at} \\ \text{each point}\end{matrix} = \begin{matrix}\text{distributed} \\ \text{load intensity} \\ \text{at each point}\end{matrix}$$

$$\begin{matrix}\text{slope of moment} \\ \text{diagram at each point}\end{matrix} = \begin{matrix}\text{Shear at} \\ \text{each point}\end{matrix}$$

Shear and Moment Diagrams

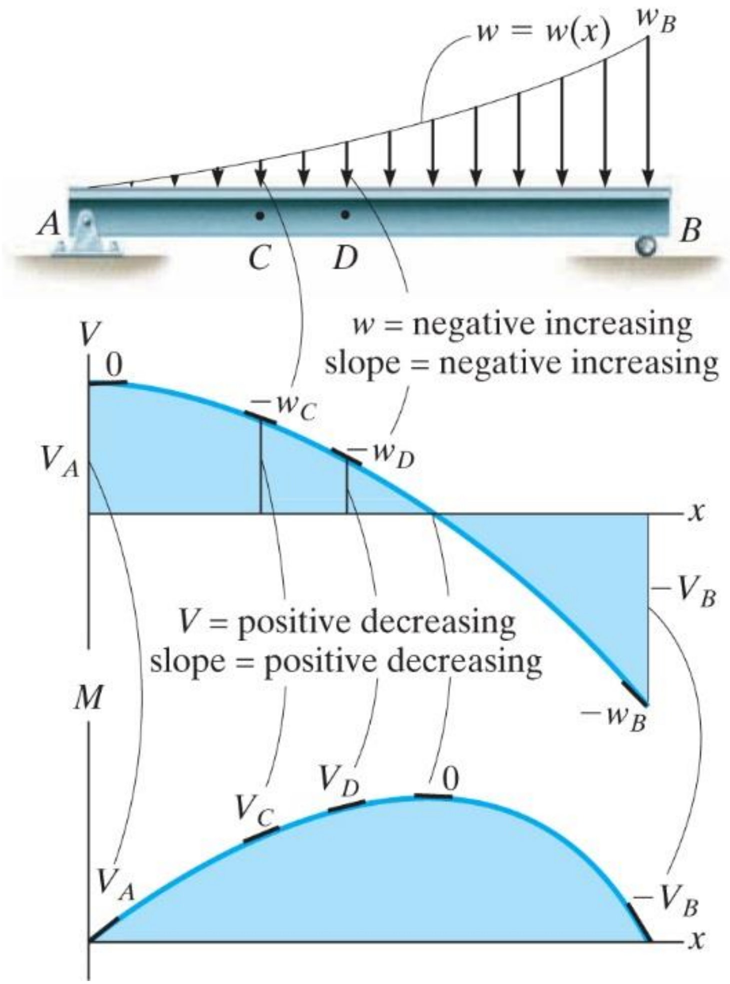
Example:

The distributed loading is negative and increases from zero to w_B

Therefore:

The shear diagram will be a curve that has a negative slope, decreasing from zero to $-w_B$

The moment diagram will then have an initial positive slope of V_A which decreases to zero, then the slope becomes negative and decreases to $-V_A$.



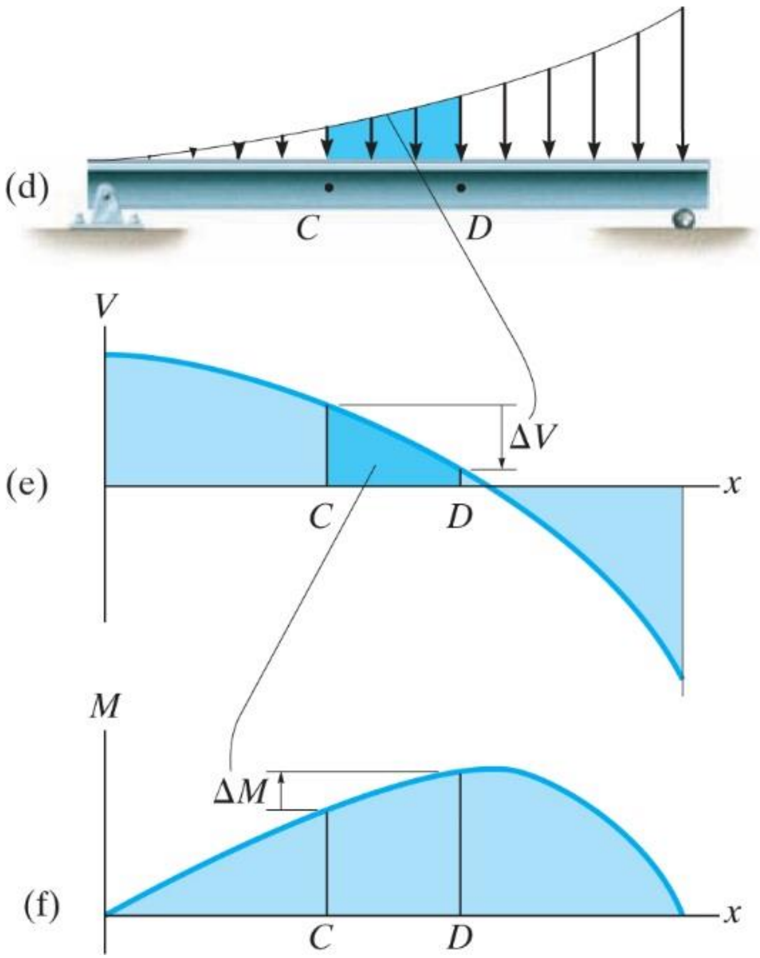
Shear and Moment Diagrams

$$\Delta V = \int w(x).dx$$

change in shear = area under distributed load

$$\Delta M = \int V.dx$$

change in moment = area under Shear diagram



Shear and Moment Diagrams

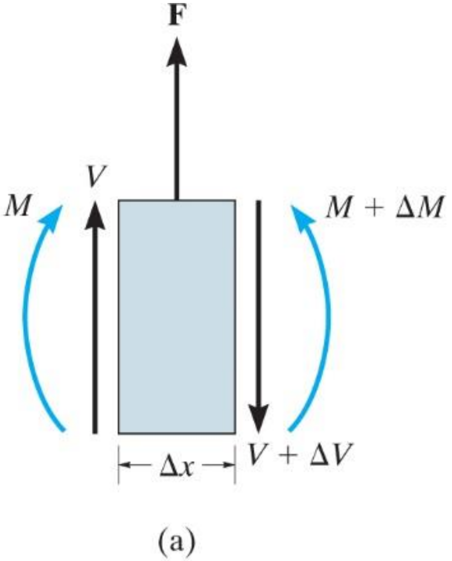
Regions of Concentrated Force and Moment

Beam under the force

$$\sum F_y = 0; \; V + F - (V + \Delta V) = 0$$

$$\Delta V = F$$

when F acts upward on the beam, ΔV is positive so the shear will "jump" upward. Likewise, if F acts downward, the jump ΔV will be downward.



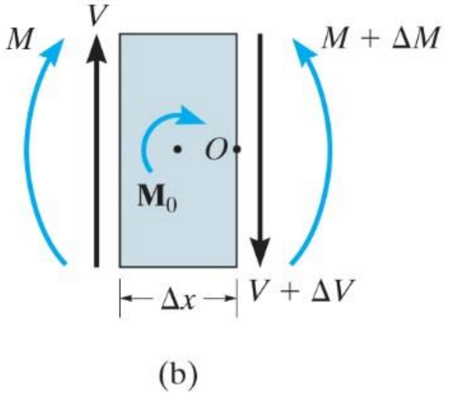
(a)

Beam segment includes the couple moment

$$\sum M_O = 0; \quad (M + \Delta M) - M_o - V \cdot \Delta x - M = 0$$

$$\Delta M = M_o$$

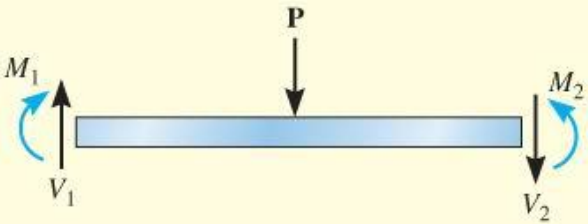
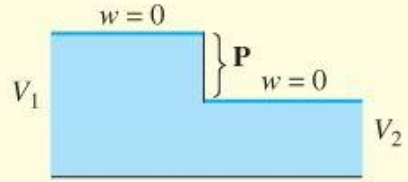
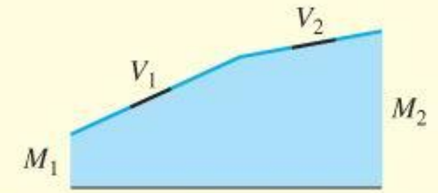
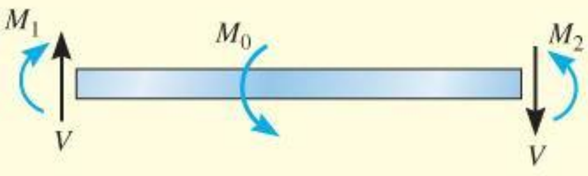
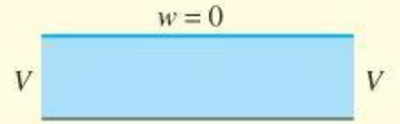
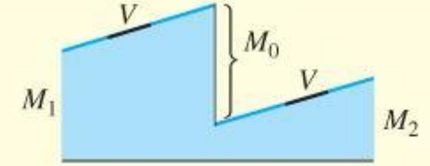
when M_0 applied clockwise, ΔM is positive so the moment diagram will "jump" upward. Likewise, when M_0 acts counterclockwise, the jump ΔM will be downward



(b)

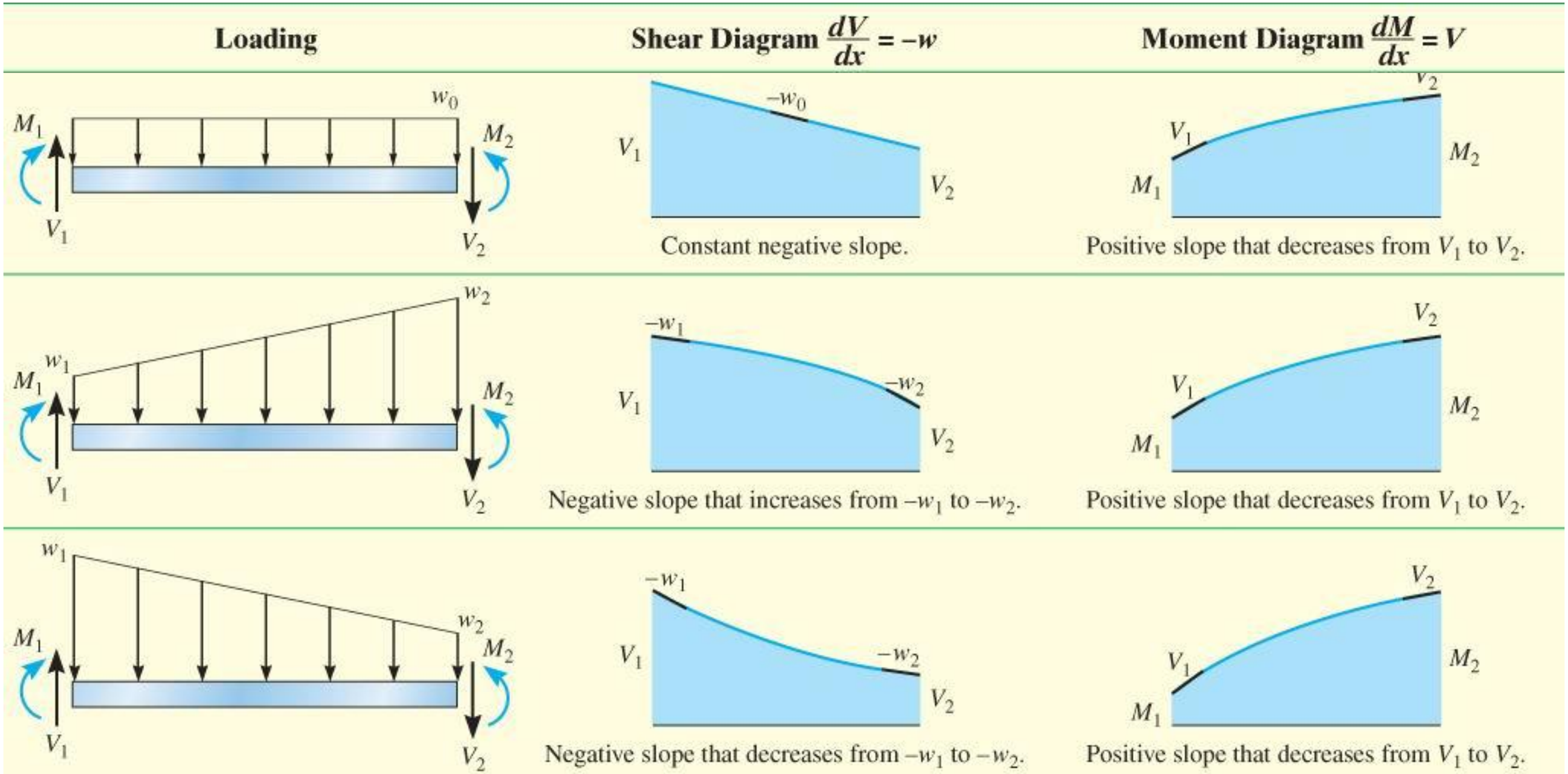
Graphical Method for Constructing Shear and Moment Diagrams

Regions of concentrated force and moment

Loading	Shear Diagram $\frac{dV}{dx} = -w$	Moment Diagram $\frac{dM}{dx} = V$
	 Downward force **P** causes V to jump downward from V_1 to V_2.	 Constant slope changes from V_1 to V_2.
	 No change in shear since slope $w = 0$.	 Constant positive slope. Counterclockwise $\mathbf{M}_0$ causes M to jump downward.

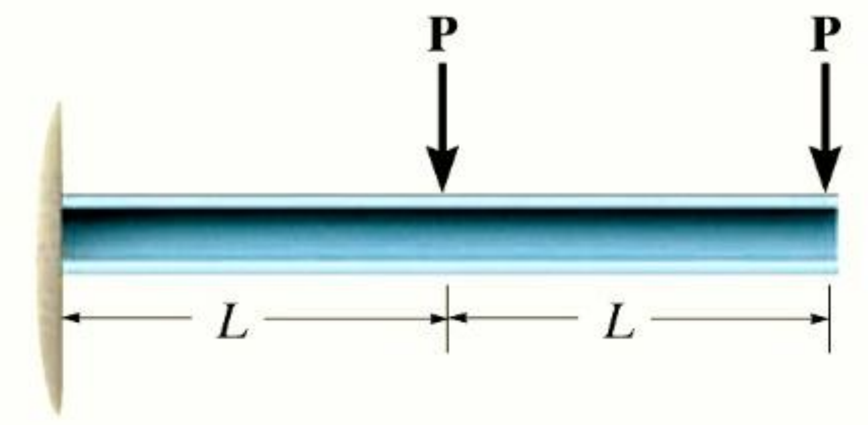
Graphical Method for Constructing Shear and Moment Diagrams

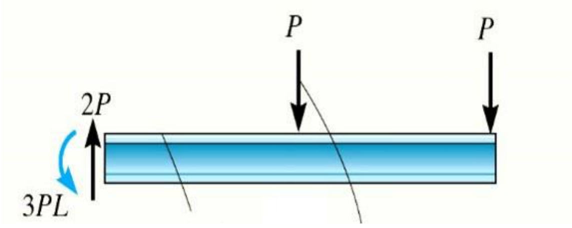
Regions of concentrated force and moment

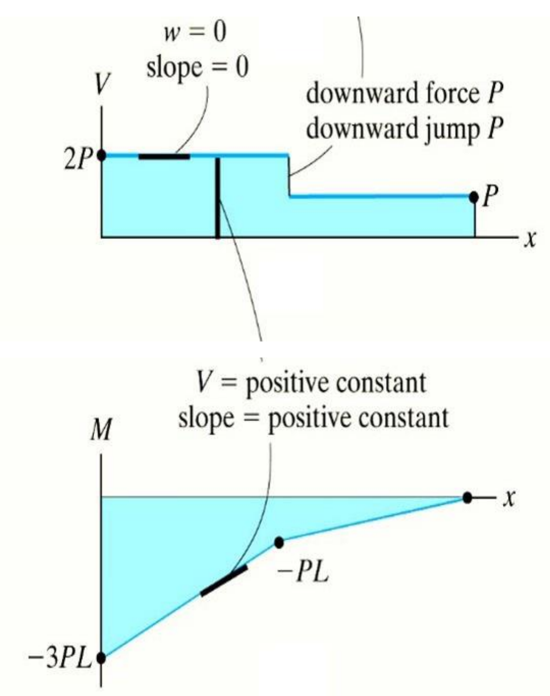


Graphical Method for Constructing Shear and Moment Diagrams

Draw the shear and moment diagrams







Graphical Method for Constructing Shear and Moment Diagrams

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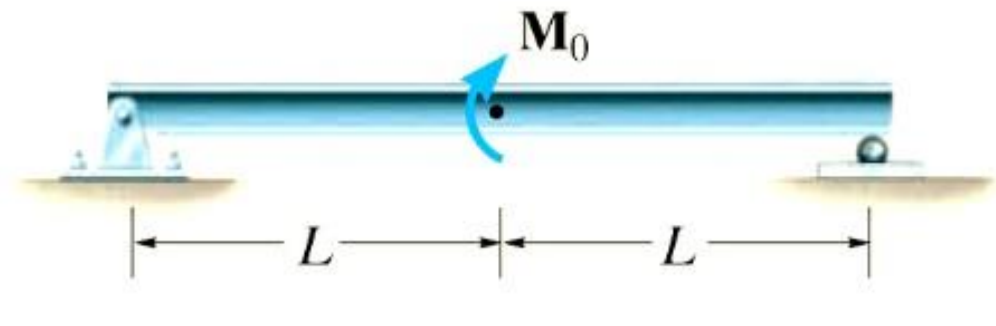
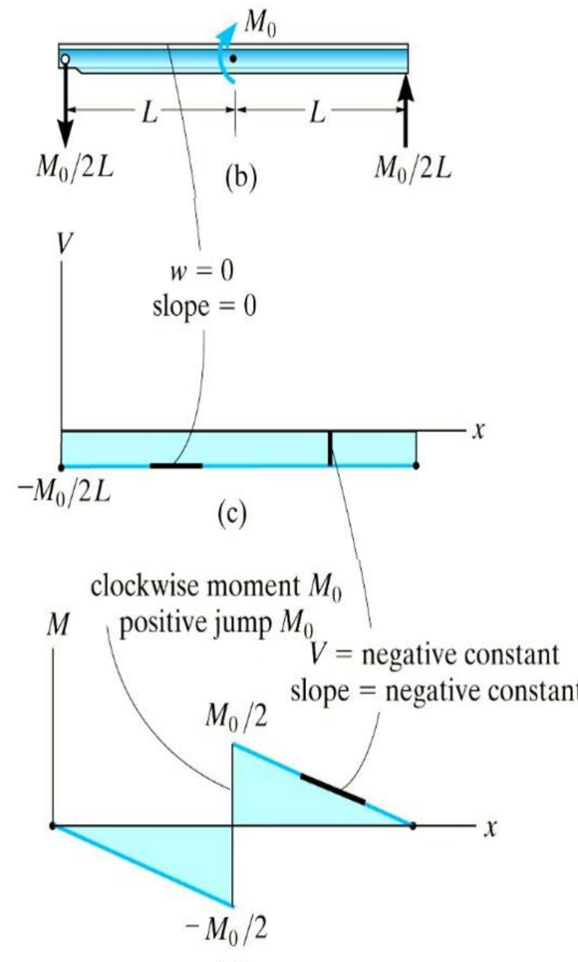


(a)



Graphical Method for Constructing Shear and Moment Diagrams

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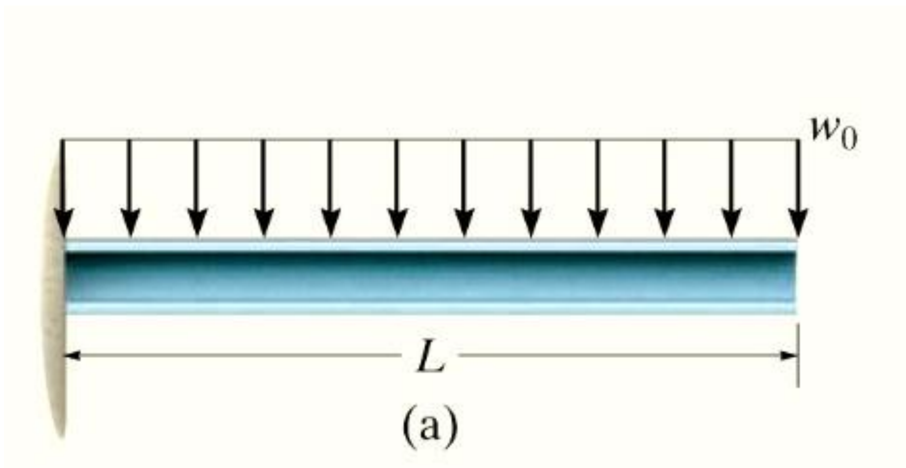

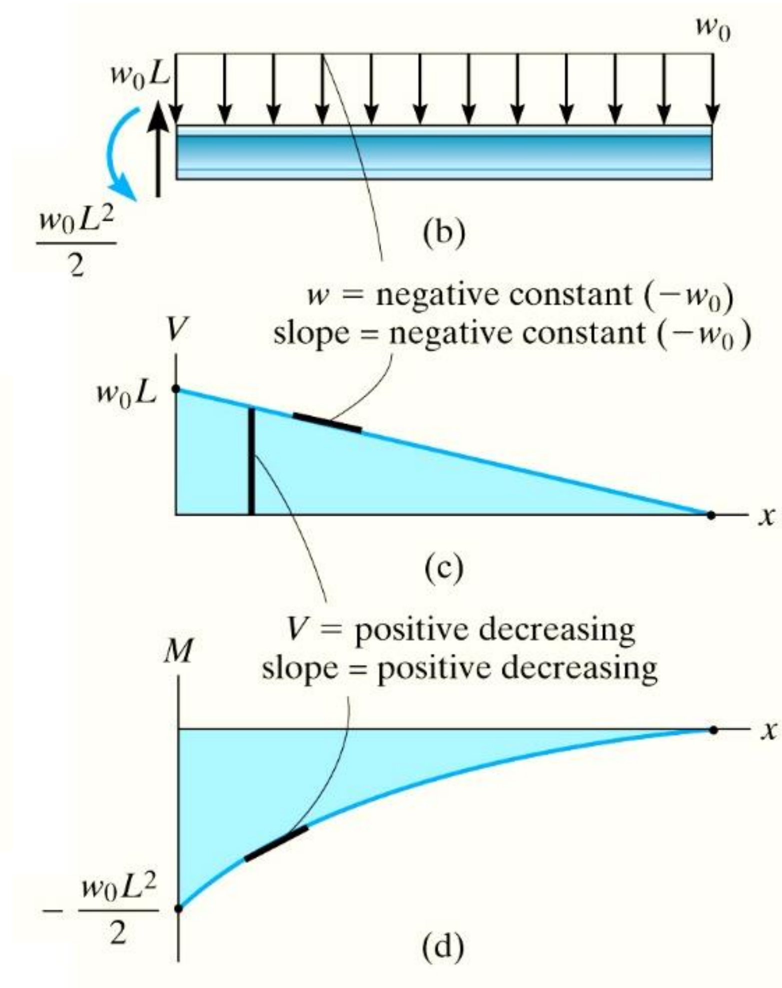

Graphical Method for Constructing Shear and Moment Diagrams

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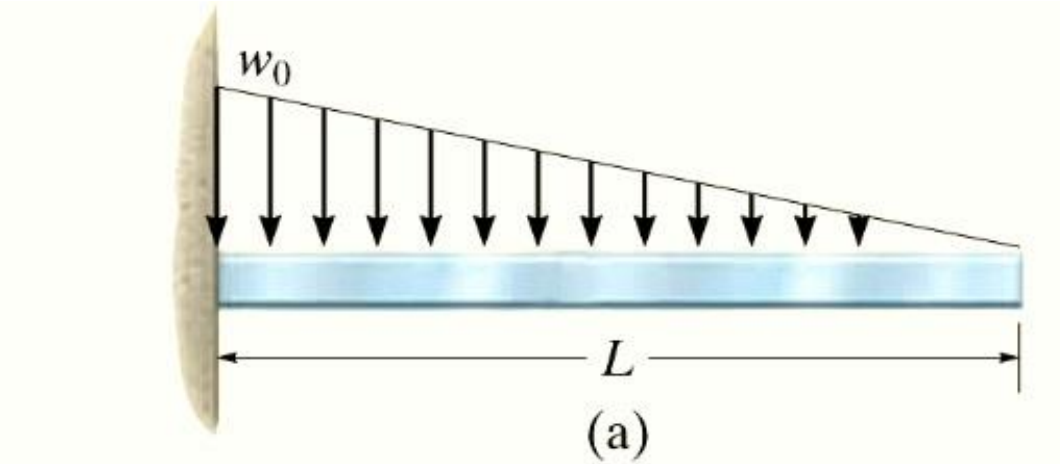
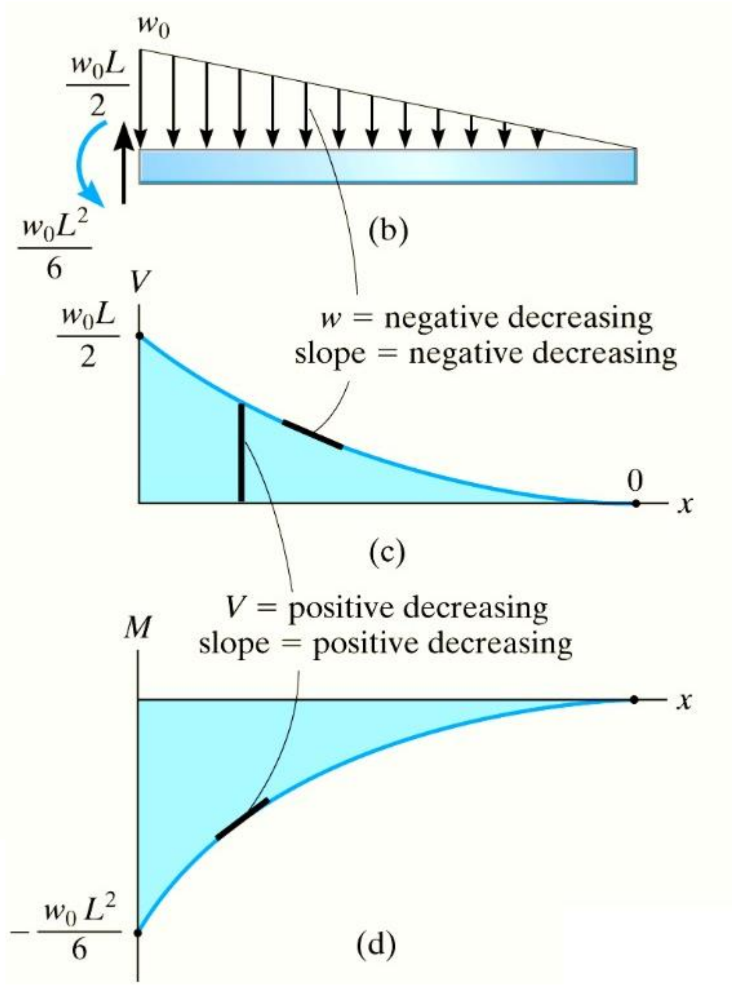
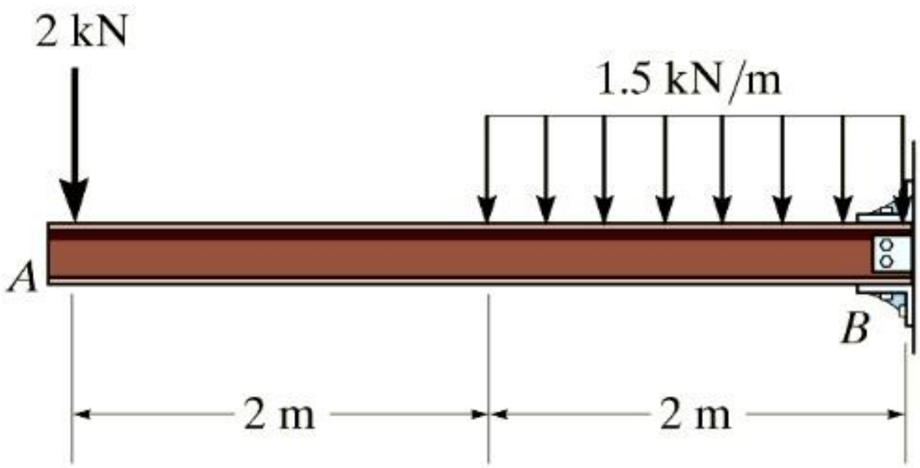
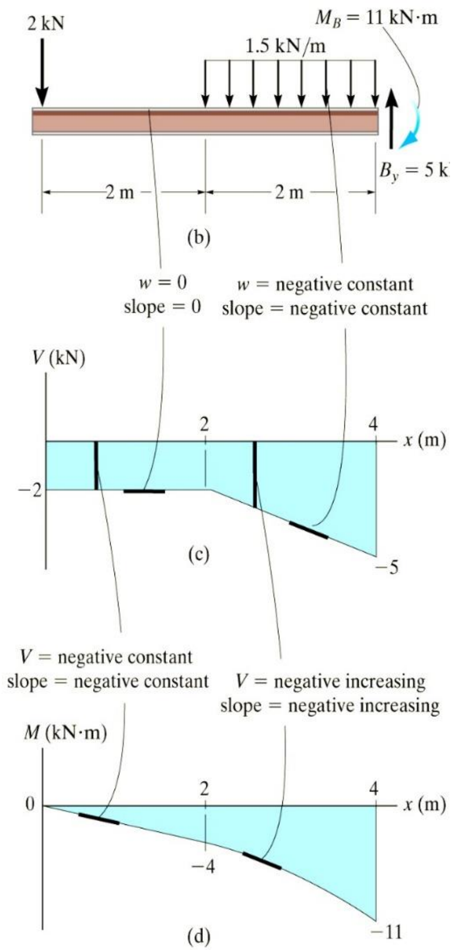




Graphical Method for Constructing Shear and Moment Diagrams

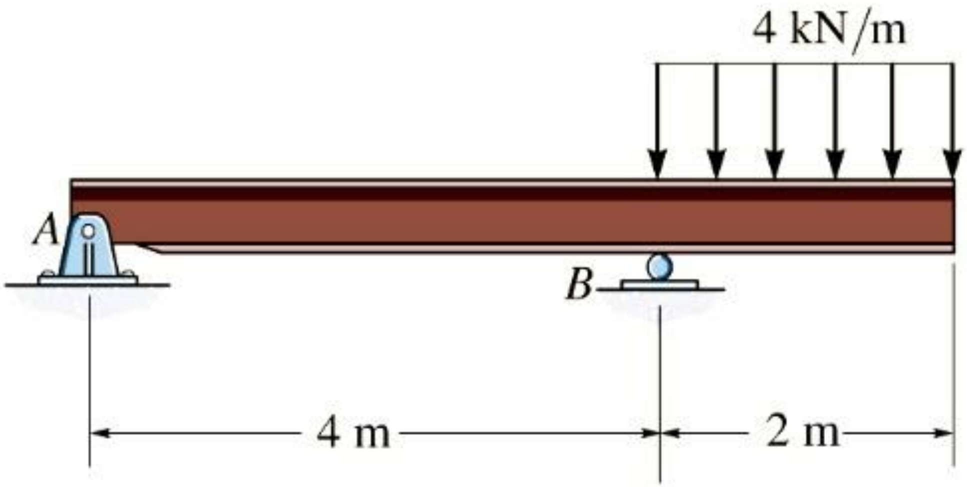
Draw the shear and moment diagrams

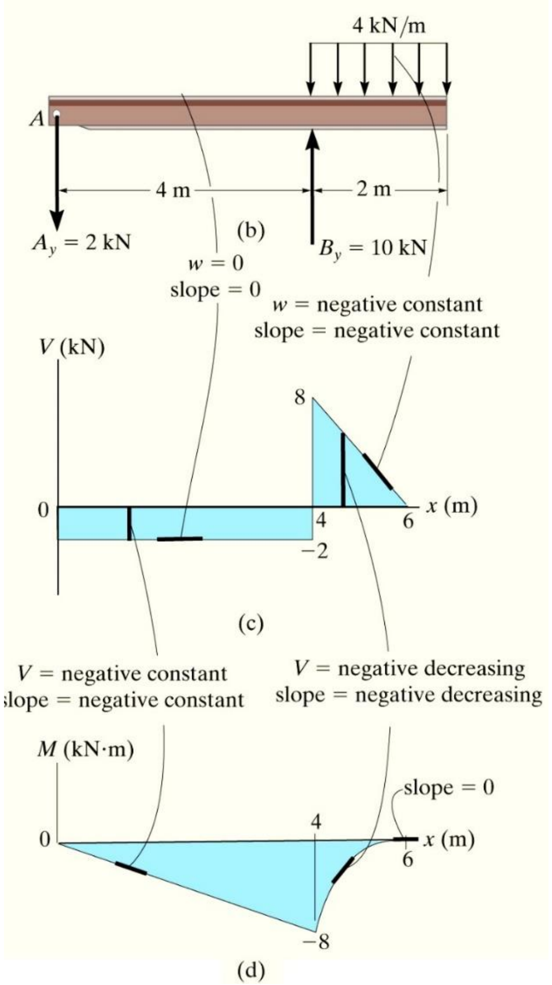




Graphical Method for Constructing Shear and Moment Diagrams

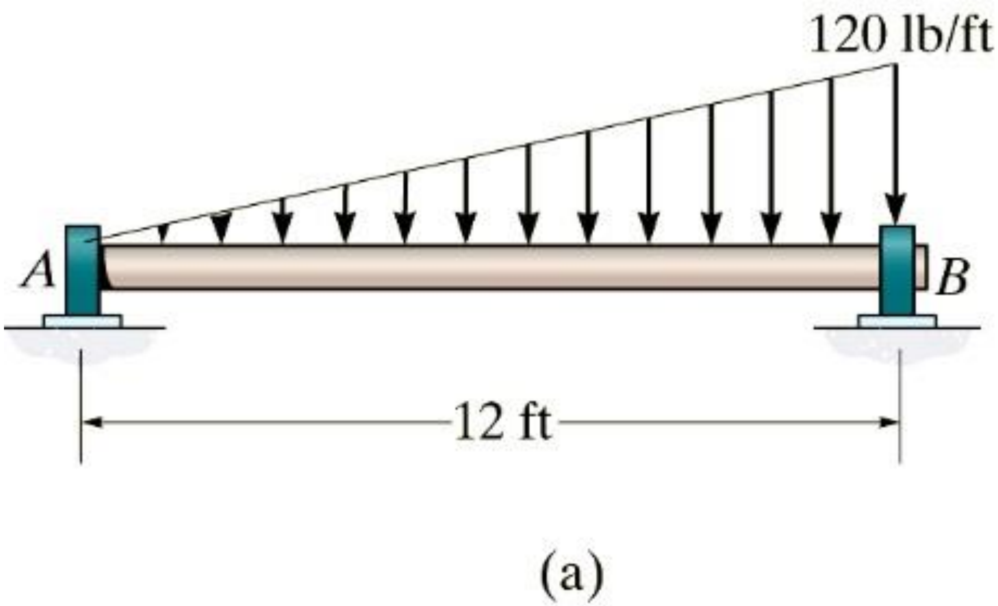
Draw the shear and moment diagrams

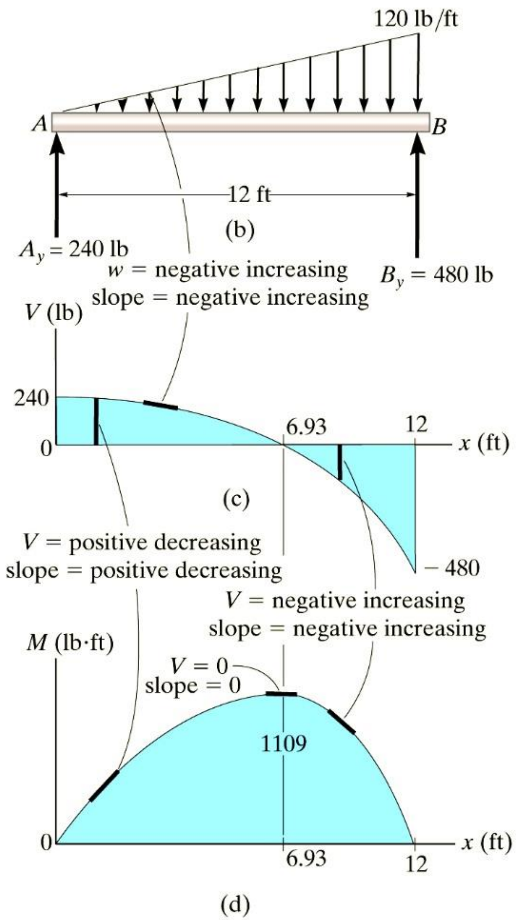




Graphical Method for Constructing Shear and Moment Diagrams

Draw the shear and moment diagrams





Graphical Method for Constructing Shear and Moment Diagrams

Draw the shear and moment diagrams