Mechanics of Materials

Lecture 3

Strain

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Lecture Objectives

- ✓ Define concept of normal strain.
- ✓ Define concept of shear strain.
- ✓ Determine normal and shear strain in engineering applications
- ✓ Understand strain measurement concept

Lecture Outline

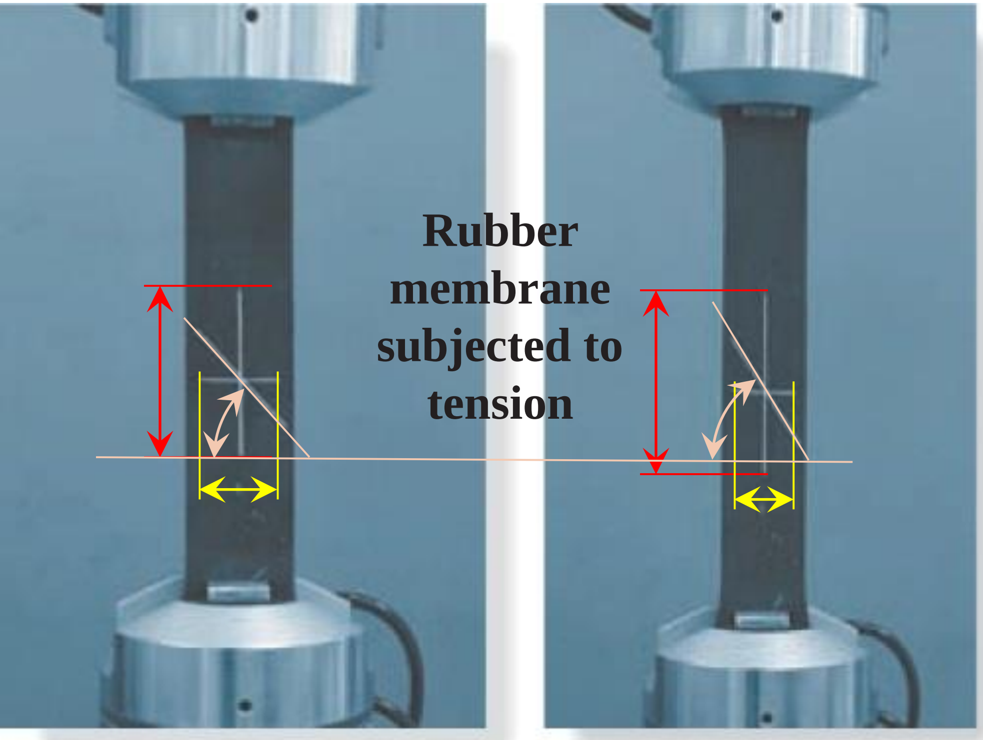
- ✓ Deformation
- ✓ Strain
- ✓ Strain Measurement Methods
- ✓ Strain Gauge

Deformation

Whenever a force is applied to a body, it will tend to change the body's shape and size. These changes are referred to as ***deformation***.

- ✓ Can be highly visible or practically unnoticeable.
- ✓ Can also occur when temperature of a body is changed.
- ✓ Is not uniform throughout a body's volume, thus change in geometry of any line segment within body may vary along its length.

Deformation



Before After

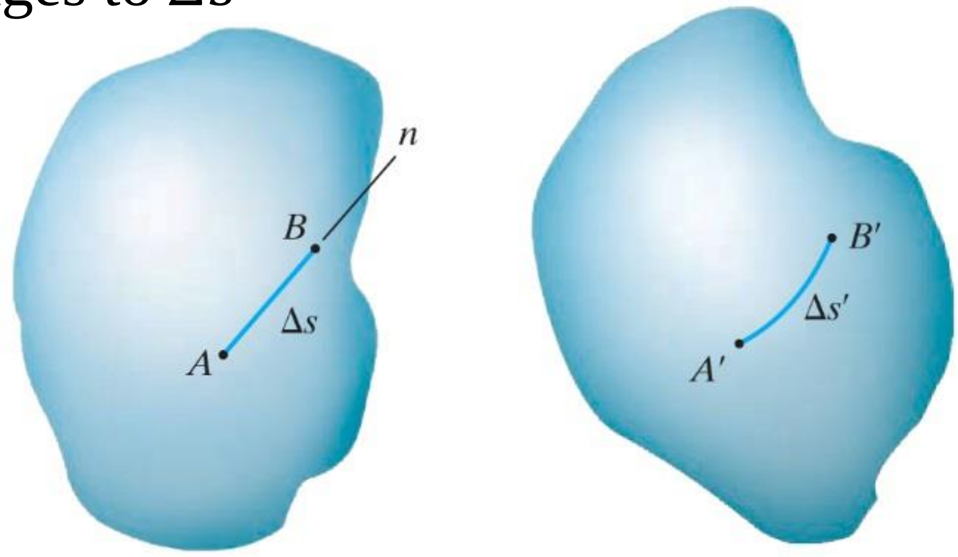
- The vertical line is lengthened.
- The horizontal line is shortened.
- The inclined line changes its length and rotates.

Strain Concept

Strain is actually measured by experiments, and once the strain is obtained we can easily determine the stress acting within the body,

Normal Strain

- ✓ Defined as the elongation or contraction of a line segment per unit of length
- ✓ Consider line AB in figure below
- ✓ After deformation, Δs changes to $\Delta s'$



Undeformed body　　　Deformed body

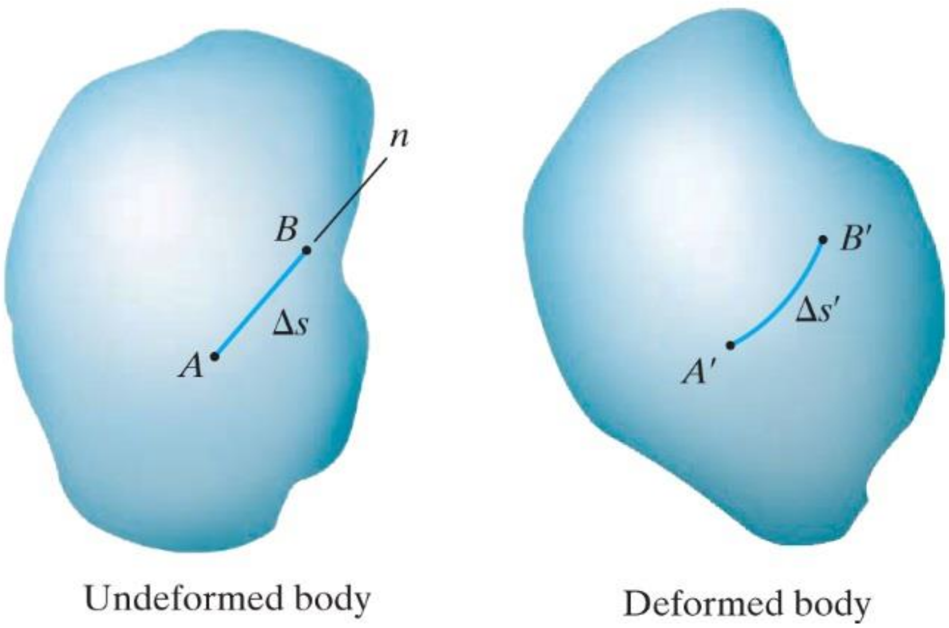
Normal Strain

$$\varepsilon = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\varepsilon = \lim_{B \to A \ along \ n} \frac{\Delta s' - \Delta s}{\Delta s}$$

$\varepsilon > 0$ Initial line will elongate

$\varepsilon < 0$ Initial line will contracts



Normal Strain

Dimensionless quantity : since it is a ratio of two lengths.

$$\varepsilon = \frac{m}{m} \qquad \varepsilon = \frac{\mu m}{m} = 10^{6}\,\frac{m}{m}$$

For experimental work, strain is expressed as a percent,

$$\varepsilon = 120.10^{-6}\, m/m = 120\,\mu m/m = 0,0120\% = 120\,\mu\varepsilon = 120 micros$$

Shear Strain

- ✓ Defined as the change in angle that occurs between two line segments that were originally perpendicular to one another
- ✓ This angle is denoted by γ (gamma) and measured in radians (rad).

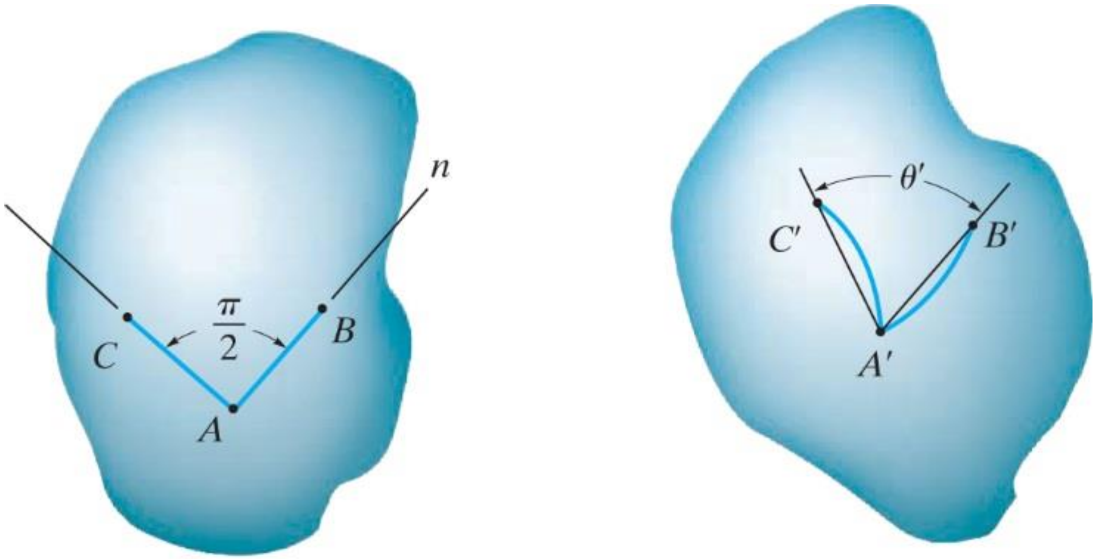
Shear Strain

Consider line segments AB and AC originating from same point A in a body, and directed along the perpendicular n and t axes
After deformation, lines become curves, such that angle between them at A is θ'

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$

$$\theta' > \frac{\pi}{2} \Rightarrow \gamma < 0$$

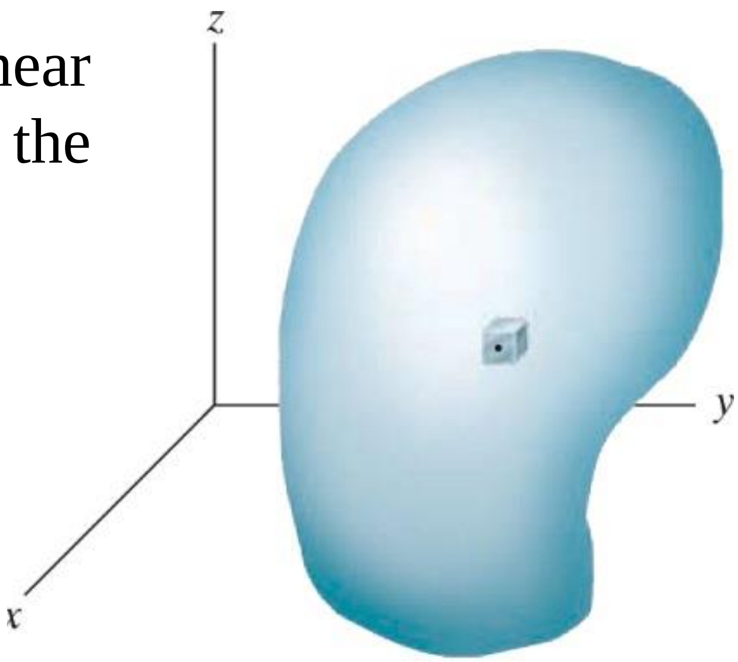
$$\theta' < \frac{\pi}{2} \Rightarrow \gamma > 0$$



Undeformed body Deformed body

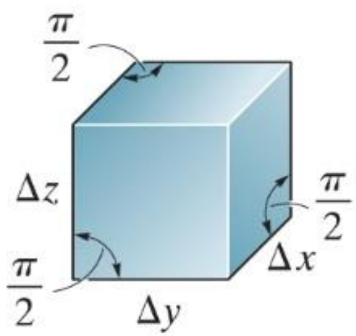
Cartesian strain components

Using above definitions of normal and shear strain, we show how they describe the deformation of the body



(a)

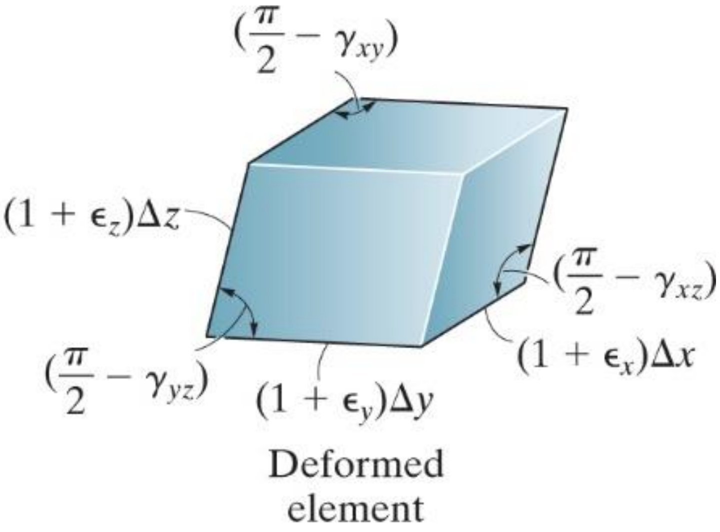
Divide body into small elements with undeformed dimensions of Δx, Δy and Δz



Undeformed element

Cartesian strain components

Since element is very small, deformed shape of element is a parallelepiped



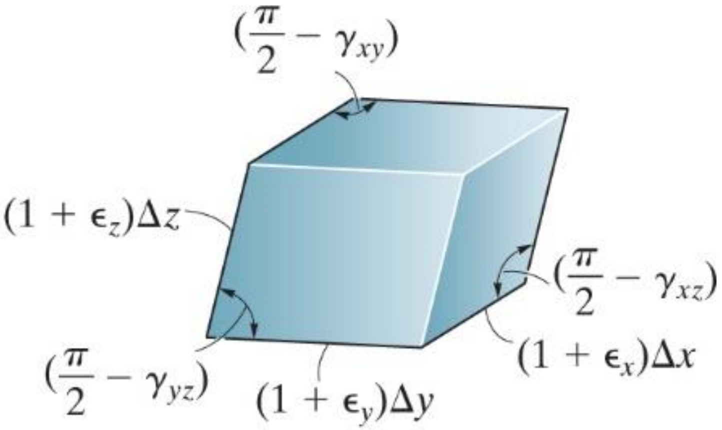
Approx. lengths of sides of parallelepiped are

$(1 + \varepsilon_x)\,\Delta x \qquad (1 + \varepsilon_y)\Delta y \qquad (1 + \varepsilon_z)\Delta z$

Cartesian strain components

Approx. angles between the sides are

$$\frac{\pi}{2} - \gamma_{xy} \qquad \frac{\pi}{2} - \gamma_{yz} \qquad \frac{\pi}{2} - \gamma_{xz}$$



Deformed element

Normal strains cause a change in its *volume*

Shear strains cause a change in its *shape*

To summarize, state of strain at a point requires specifying 3 normal strains; ε_x, ε_y, ε_z and 3 shear strains of γ_{xy}, γ_{yz}, γ_{xz}

Small Strain Analysis

Most engineering design involves applications for which only *small deformations* are allowed, therefore, we will assume that the deformations that take place within a body are almost infinitesimal. In particular, the *normal strains* occurring within the material are *very small* compared to 1.

This assumption is widely applied in practical engineering problems, and is referred to as **small strain analysis**

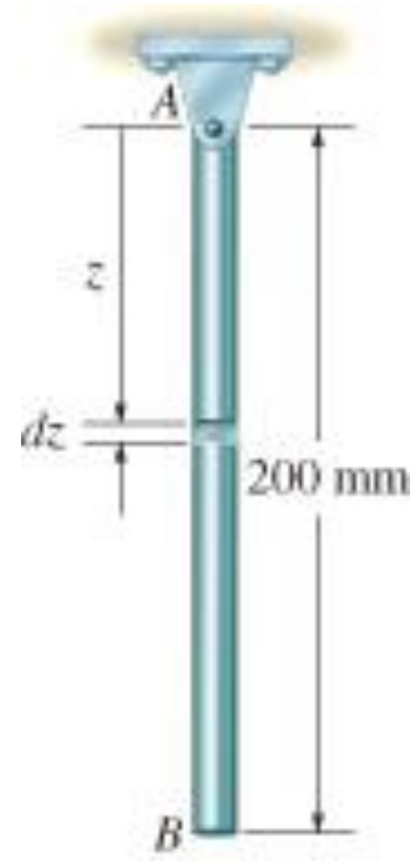
Example 1

Rod below is subjected to temperature increase along its axis, creating a normal strain of $\varepsilon_z = 40(10^{-3})z^{1/2},$ where z is given in meters.

Determine:

(a) displacement of end B of rod due to temperature increase,

(b) average normal strain in the rod.



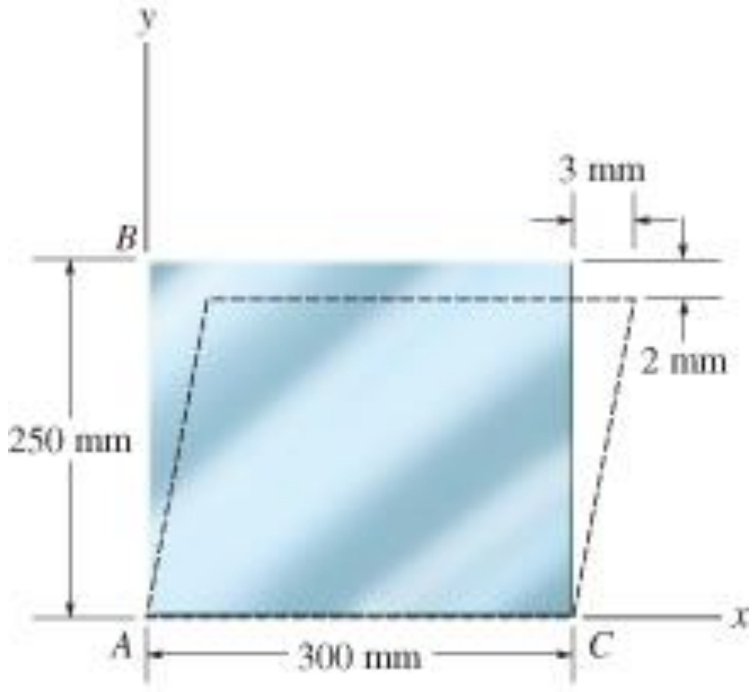
Example 2

Plate is deformed as shown in figure. In this deformed shape, horizontal lines on the on plate remain horizontal and do not change their length.

Determine

(a) average normal strain along side *AB*,

(b) average shear strain in the plate relative to *x* and *y* axes



(a)

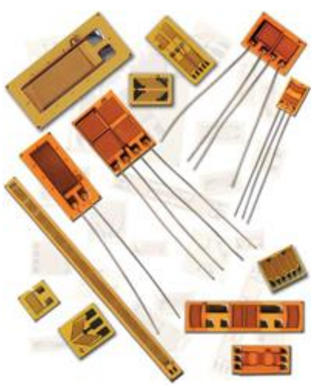
Strain Measurement Methods

Mechanical

Strain Measurement

Electrical

Optical



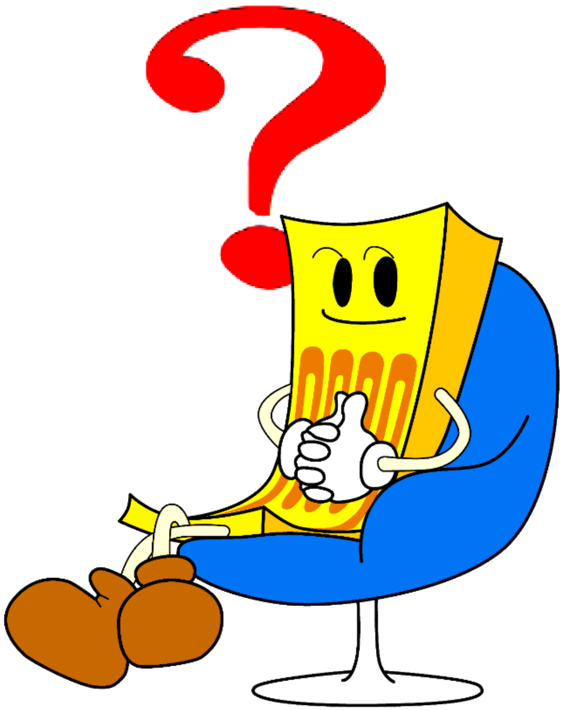
Strain Measurement Methods

1. Mechanical
2. Moiré technique
3. Interferometric strain gages
4. Electric strain gages
5. Brittle coatings
6. Photoelasticity
7. X-ray diffraction
8. Holographs
9. Laser speckle interferometry

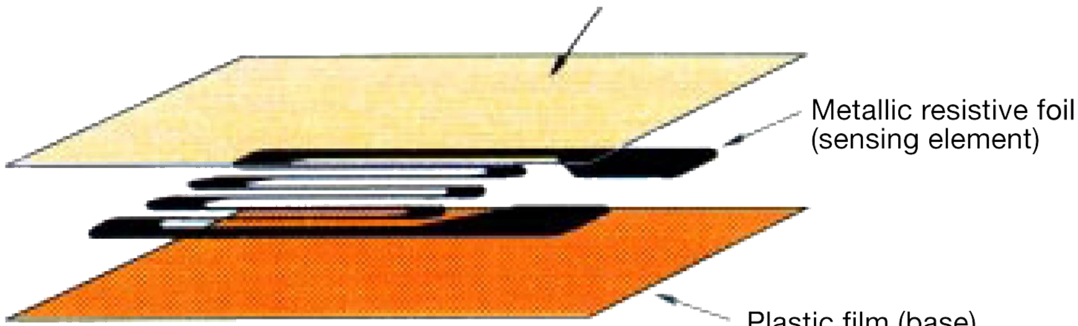
Strain Gauge

What is a Strain Gauge ?

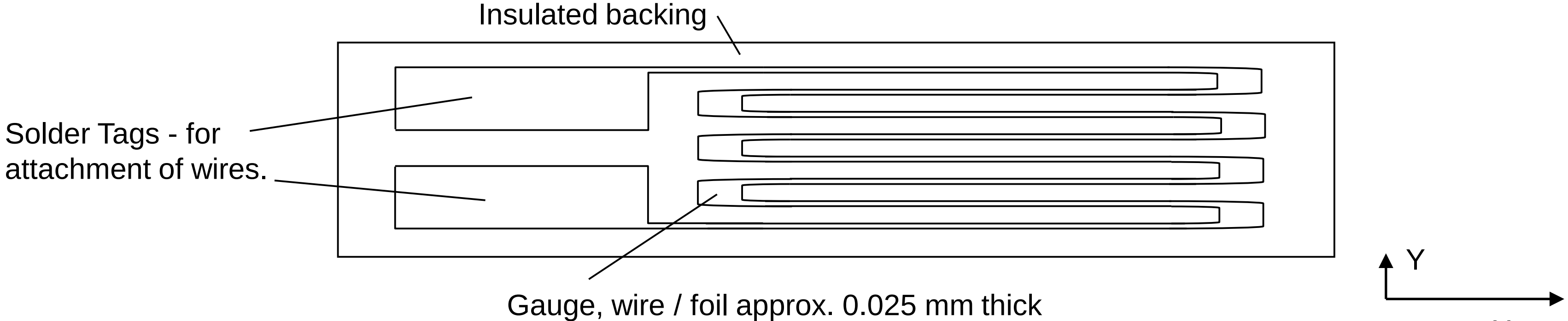
1. Strain Gauge is a device used to measure deformation (strain) of an object.
2. Strain gauges have been developed for the accurate measurement of strain
3. Fundamentally, all strain gauges are designed to convert mechanical motion into an electronic signal.



Strain Gauge

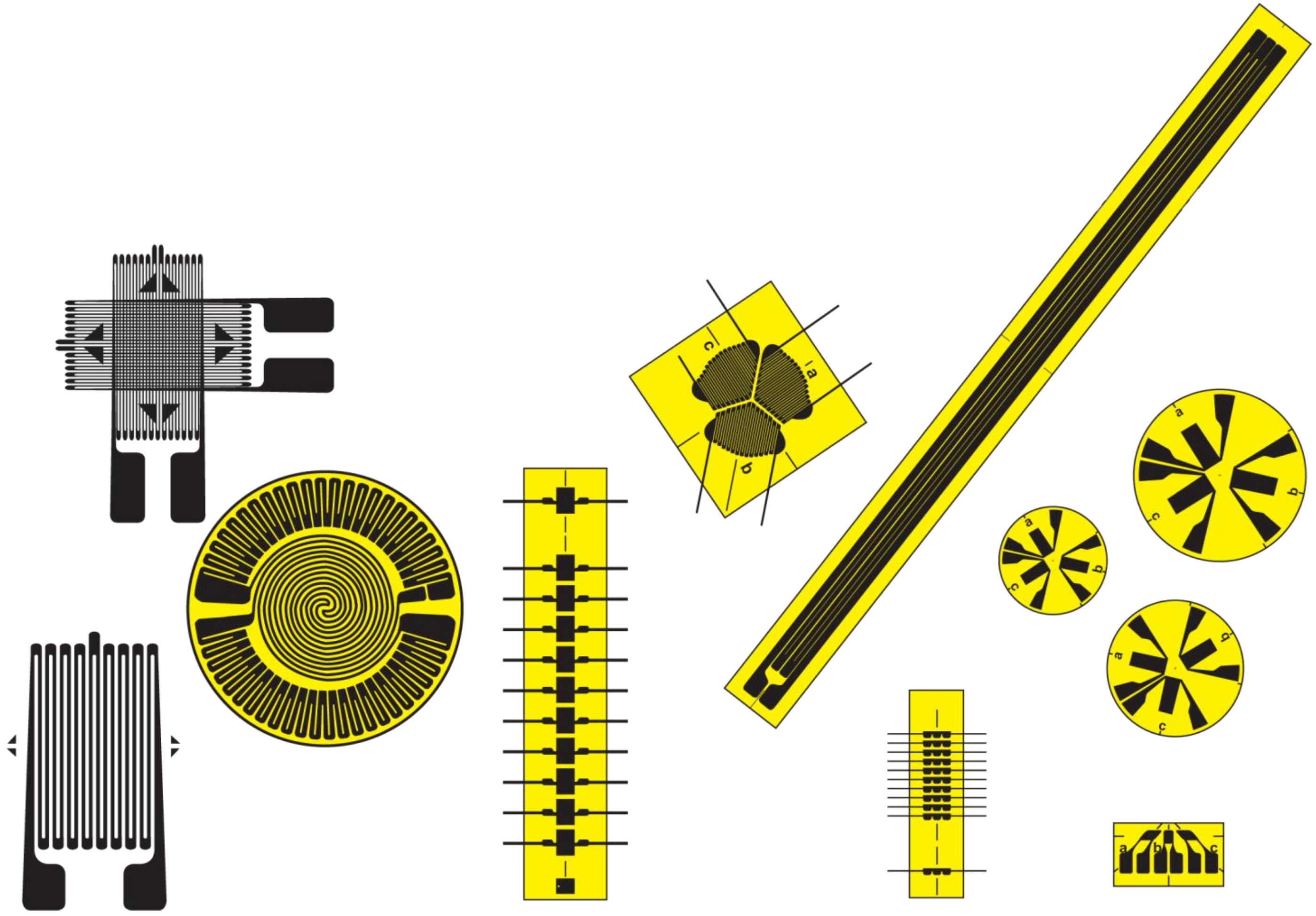


Schematic View of Strain Gauge



The gauge shown here is primarily sensitive to strain in the X direction, as the majority of the wire length is parallel to the X axis.

Strain Gauge



Strain Gauge

Strain gages in the aerospace industry
Experimental stress analysis on an aerospace component

Strain Gauge

SG bridge with integrated compensating elements on a double bending beam

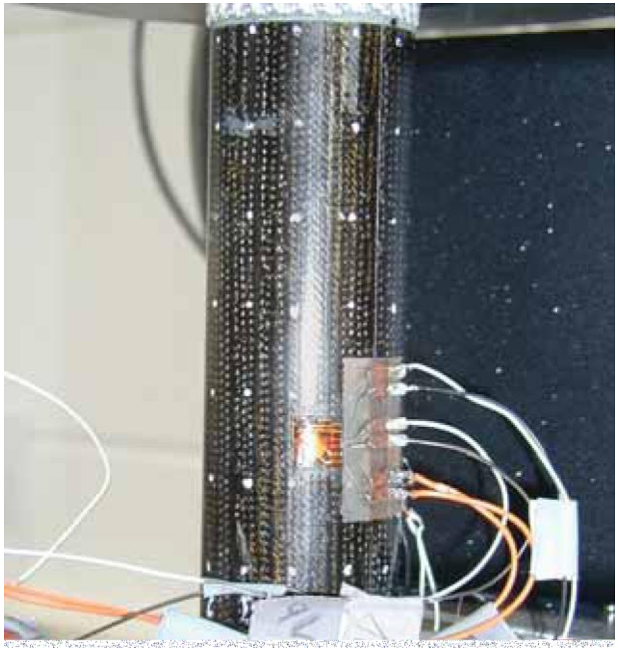
Strain gages in the manufacture of transducers

SG installed on a customized torque transducer

Strain Gauge

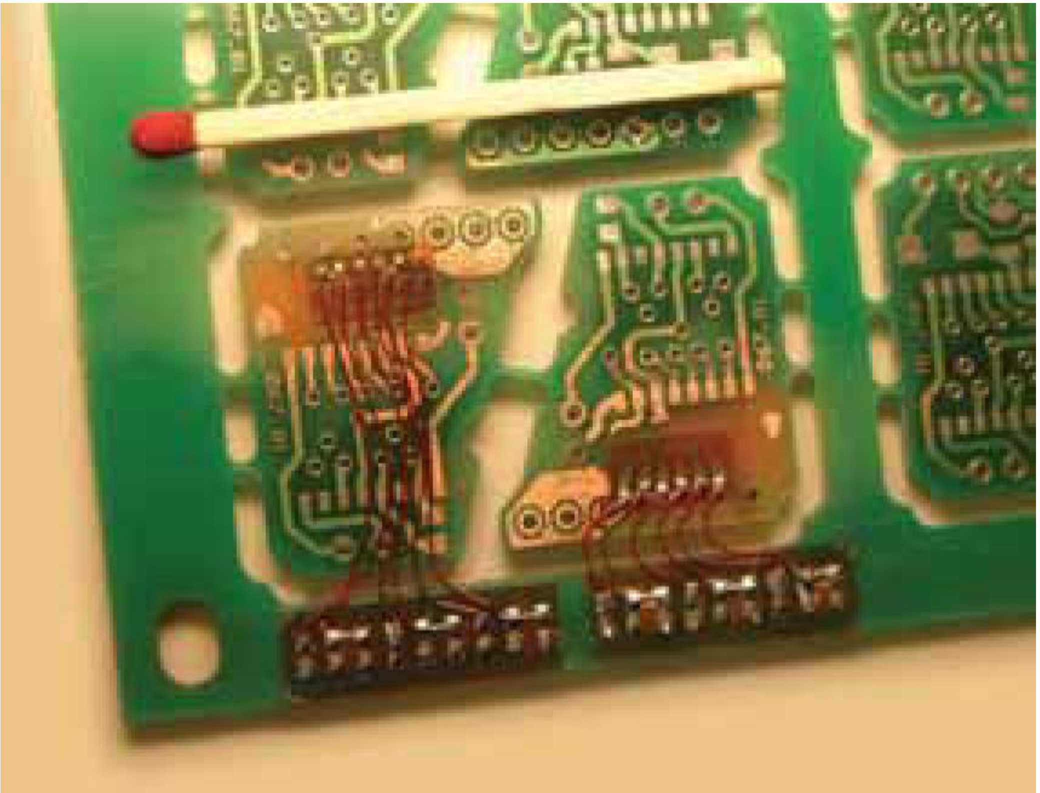
Strain gages in railway engineering
Experimental stress analysis on a railway car

Strain Gauge



Pipe specimen made of carbon-fiber reinforced plastic in a torsion fracture test

Strain Gauge



Strain gages in the electrical industry
For monitoring of the loads during the punching of PCBs

Strain Gauge

Strain Gauge

Strain Gauge

Pam Thurber; Vermont Agency of Transportation (VTrans)

Strain Gauge

Strain Gauge