Mechanics of Materials

Lecture 2

Stress

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Concept of stress

Material properties

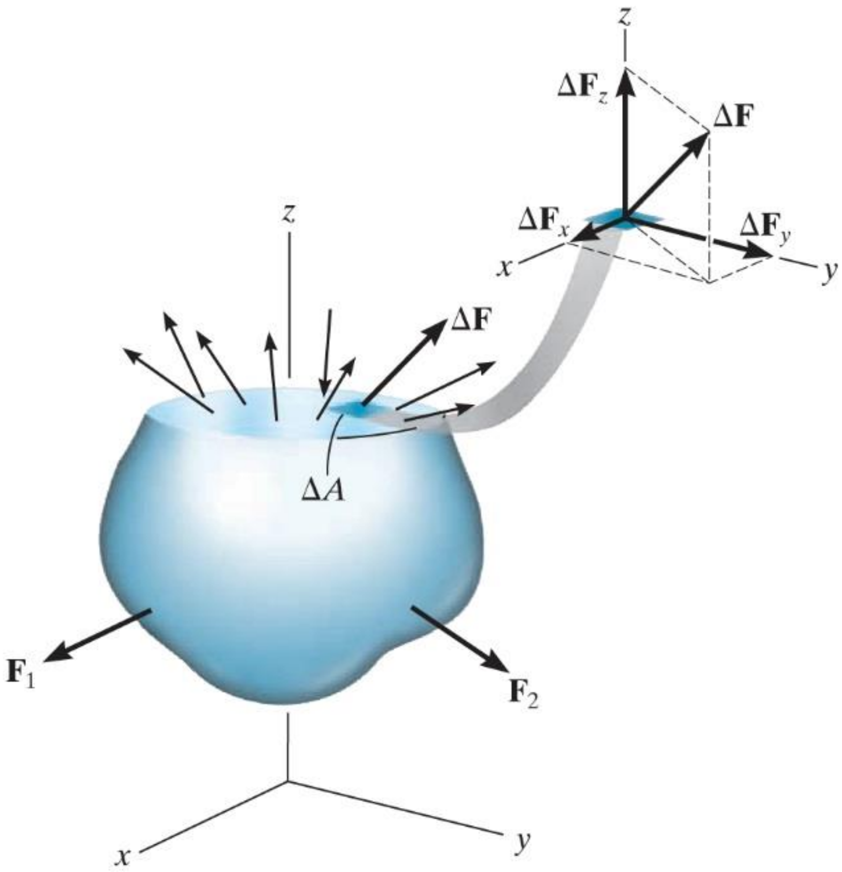
Continuous

To consist of a continuum or uniform distribution of matter having no voids.

Cohesive

All portions of matter are connected together, without having breaks, cracks, or separations

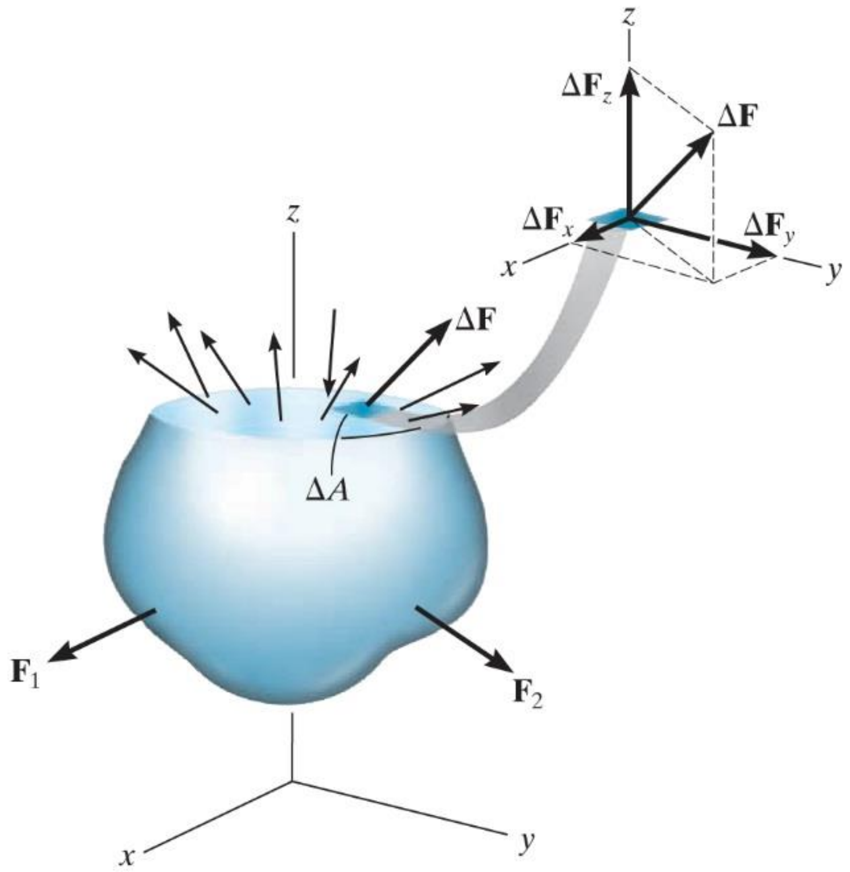
Concept of stress



ΔA Sectioned area to be subdivided into small areas

ΔF A typical finite very small force acting on the area

Concept of stress



The Force can be replaced by its three components:

ΔF_x Tangent to the area.

ΔF_y Tangent to the area.

ΔF_z Normal to the area.

Concept of stress

The quotient of the force and area

$$\lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

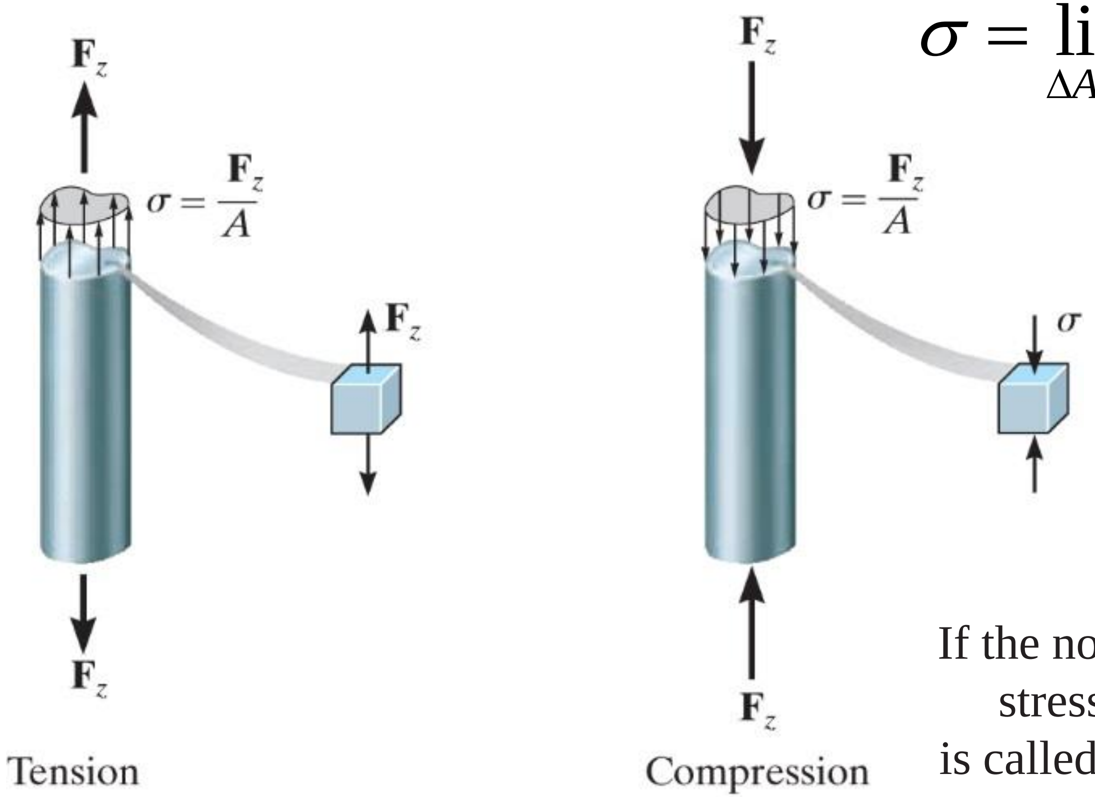
It describes the intensity of the internal force acting on a specific plane (area) passing through a point.

As the area approaches zero, so do the force and its components, this quotient will, in general, approach a finite limit.

Normal Stress

The *intensity* of the force acting normal to the area

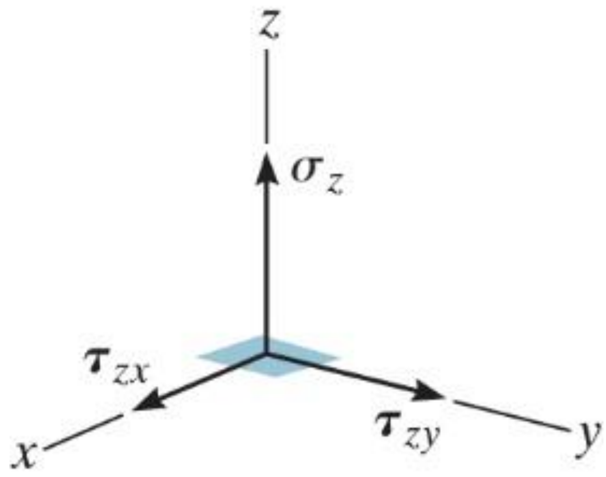
$$\sigma = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$



If the normal force or stress "pulls" is referred to as *tensile stress*,

If the normal force or stress "pushes" is called *compressive stress*.

Normal Stress



$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

z

specifies the
orientation of the area

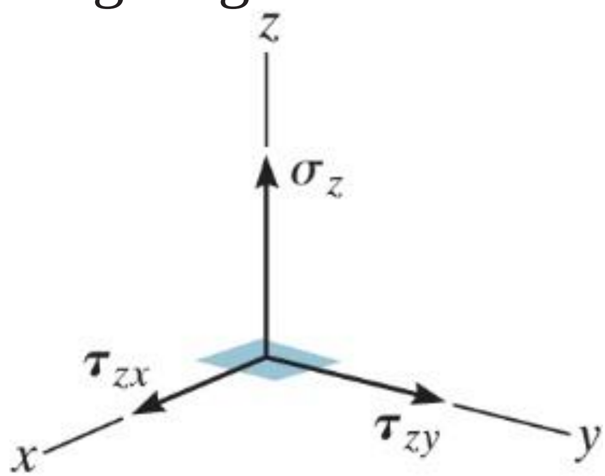
indicate the axes along
which Normal stress acts.

Shear Stress

The intensity of force acting tangent to the area

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$



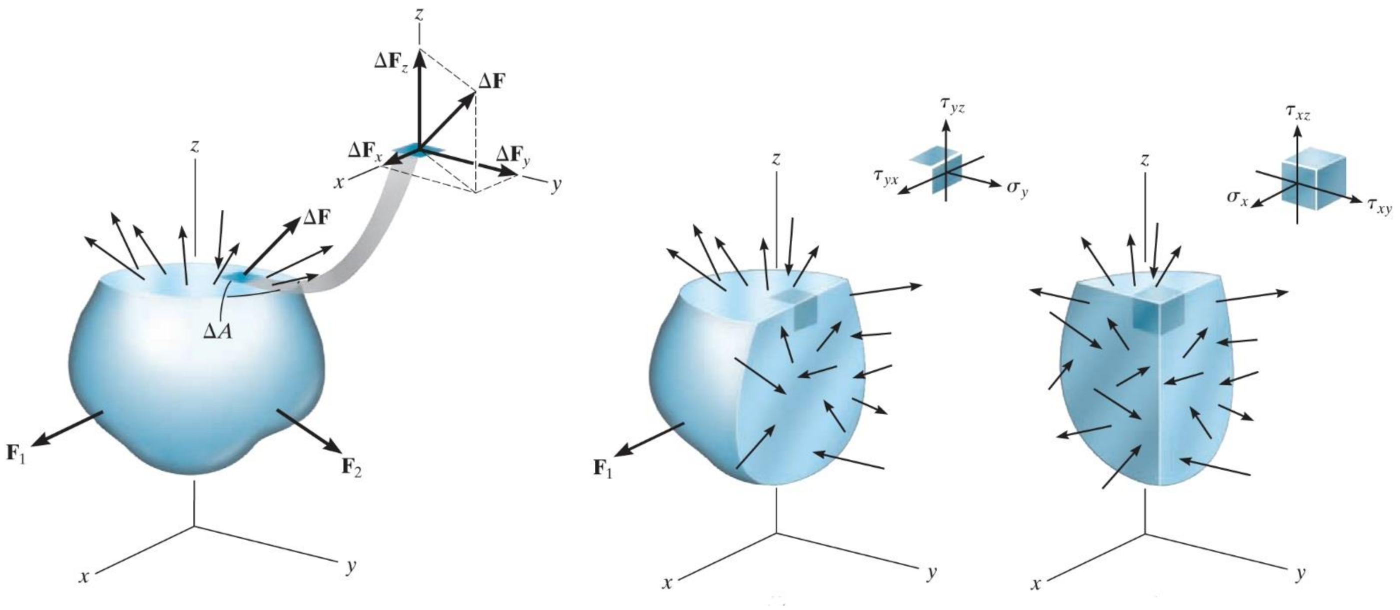
x — indicate the axes along which shear stress acts.

z — specifies the orientation of the area

y — indicate the axes along which shear stress acts.

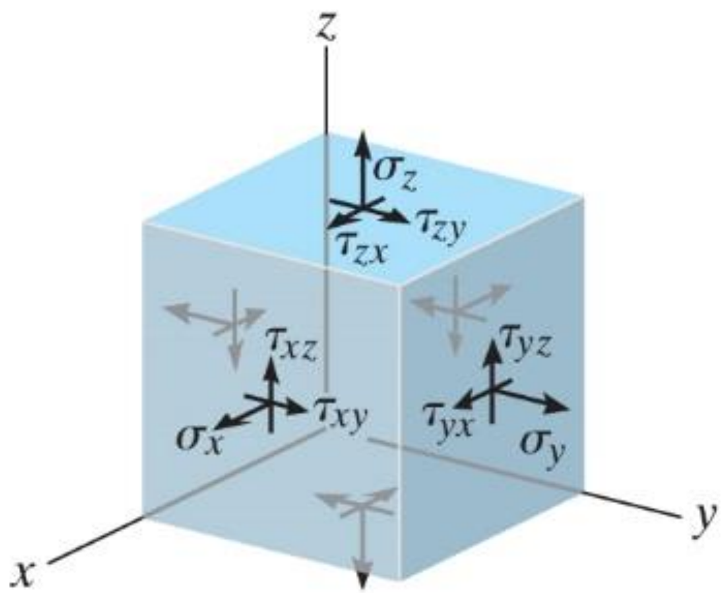
State of stress

If the body is further sectioned by planes parallel to the *xz* plane, and the *y*–*z* plane, we can then "cut out" a cubic volume element of material that represents the ***state of stress*** acting around the chosen point in the body.



State of stress

This state of stress is then characterized by three components acting on each face of the element.



Stress

Units:

Since stress represents a force per unit area, in the International Standard or SI system, the magnitudes of both normal and shear stress are specified in the basic units of newtons per square meter This unit, called a pascal is rather small

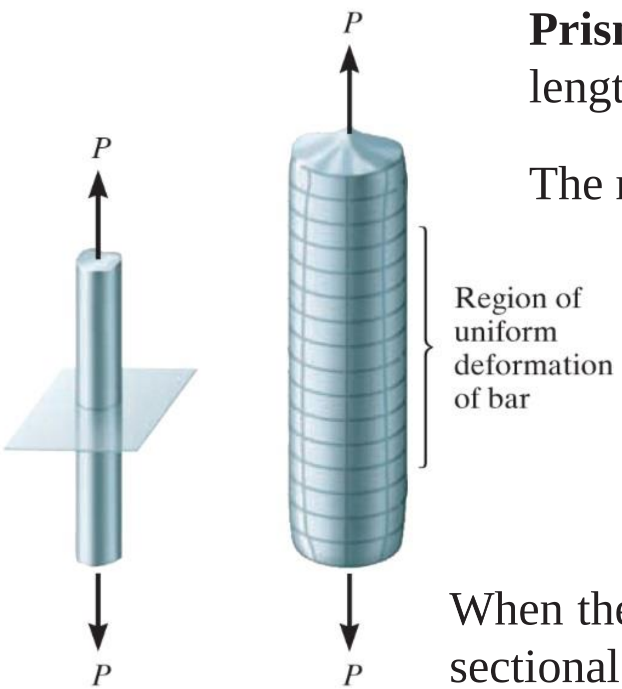
$$\left[N/m^2\right] = Pa$$

$$\left[N/mm^2\right] = MPa$$

Average Normal Stress

Bar

Axially loaded



Prismatic : since all cross sections are the same throughout its length

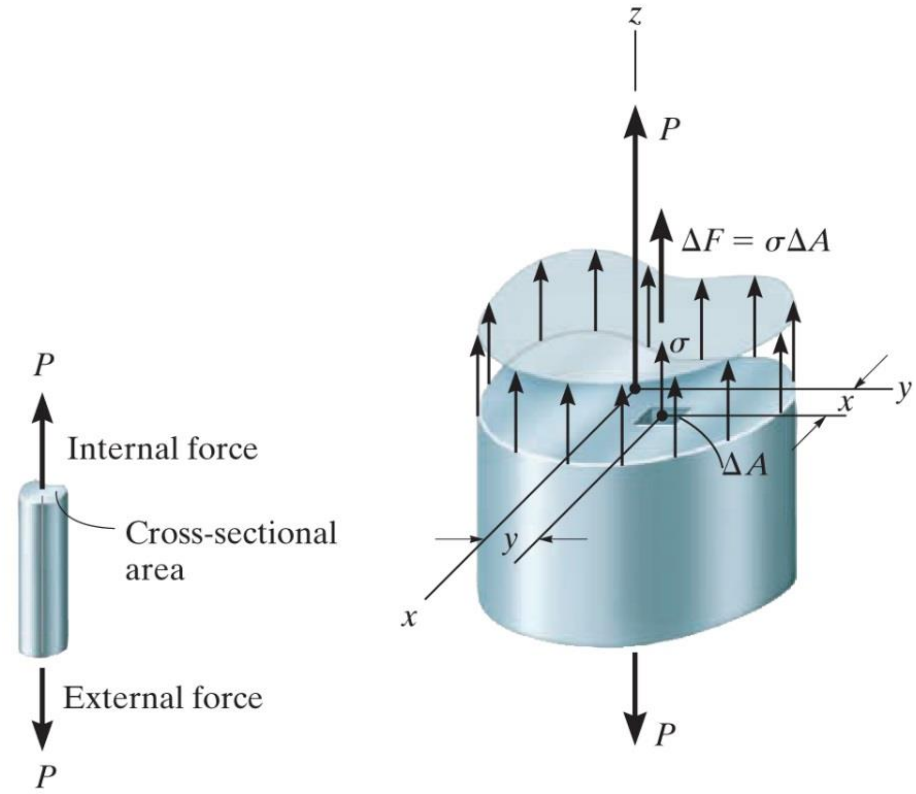
The material of the bar is both:

- ***Homogeneous material*** has the same physical and mechanical properties throughout its volume,
- and ***isotropic material*** has these same properties in all directions.

When the load P is applied to the bar through the centroid of its cross-sectional area, then the bar will deform uniformly throughout the central region of its length.

Average Normal Stress

If we pass a section through the bar, and separate it into two parts, then equilibrium requires the resultant normal force at the section to be *P*, Due to the *uniform* deformation of the material, it is necessary that the cross section be subjected to a *constant normal stress distribution.*



Average Normal Stress

As a result, each small area ΔA on the cross section is subjected to a force ΔF = σ ΔA, and the *sum* of these forces acting over the entire cross-sectional area must be equivalent to the internal resultant force **P** at the section.

$$\int dF = \int_A \sigma . dA \Rightarrow P = \sigma . A \Rightarrow \sigma = \frac{P}{A}$$

Here

σ average normal stress at any point on the cross-sectional area

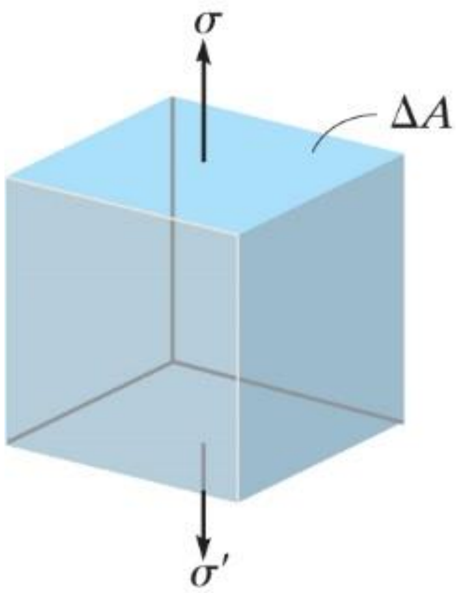
P *internal resultant normal force*, which acts through the *centroid* of the cross-sectional area. *P* is determined using the method of sections and the equations of equilibrium

A cross-sectional area of the bar where is determined

Average Normal Stress

Equilibrium. It should be apparent that only a normal stress exists on any small volume element of material located at each point on the cross section of an axially loaded bar. If we consider vertical equilibrium of the element, then apply the equation of force equilibrium,

$$\sum F_z = 0 \Rightarrow \sigma . \Delta A - \sigma' . \Delta A = 0 \Rightarrow \sigma = \sigma'$$



In other words, the two normal stress components on the element must be equal in magnitude but opposite in direction. This is referred to as *uniaxial stress*.

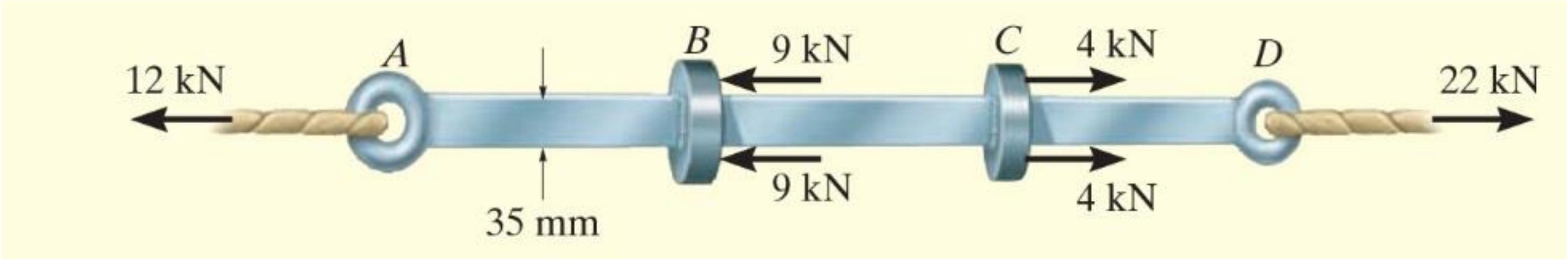
Maximum Average Normal Stress.

$$\left.\begin{matrix} P \\ A \end{matrix}\right\} = const. \Rightarrow \sigma = const.$$

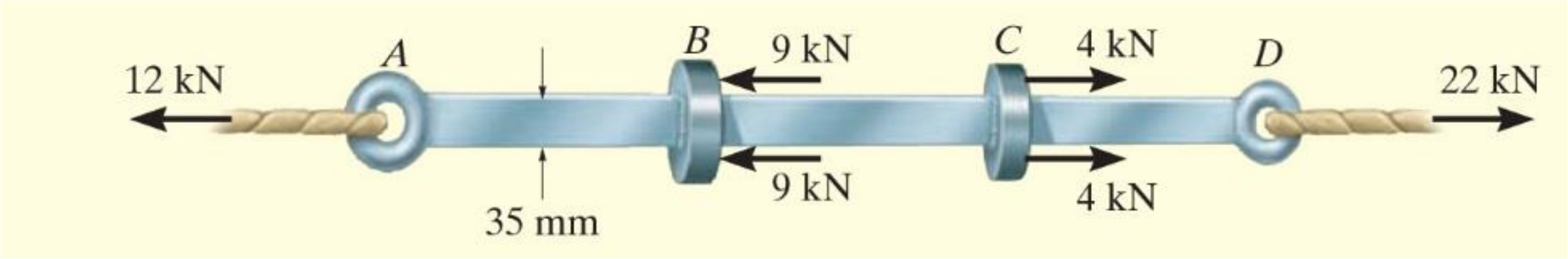
Generally the bar may be subjected to *several* external loads along its axis, or a change in its cross-sectional area may occur. As a result, the normal stress within the bar could be different from one section to the next, and, if the *maximum* average normal stress is to be determined, then it becomes important to find the location where the ratio P/A is a *maximum*. To do this it is necessary to determine the internal force P at various sections along the bar. Here it may be helpful to show this variation by drawing an ***axial or normal force diagram***. Specifically, this diagram is a plot of the normal force P versus its position x along the bar's length. As a sign convention, P will be positive if it causes tension in the member, and negative if it causes compression. Once the internal loading throughout the bar is known, the maximum ratio of P/A can then be identified.

Example 1

The bar below has a constant width of 35mm and a thickness of 10mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



Example 1



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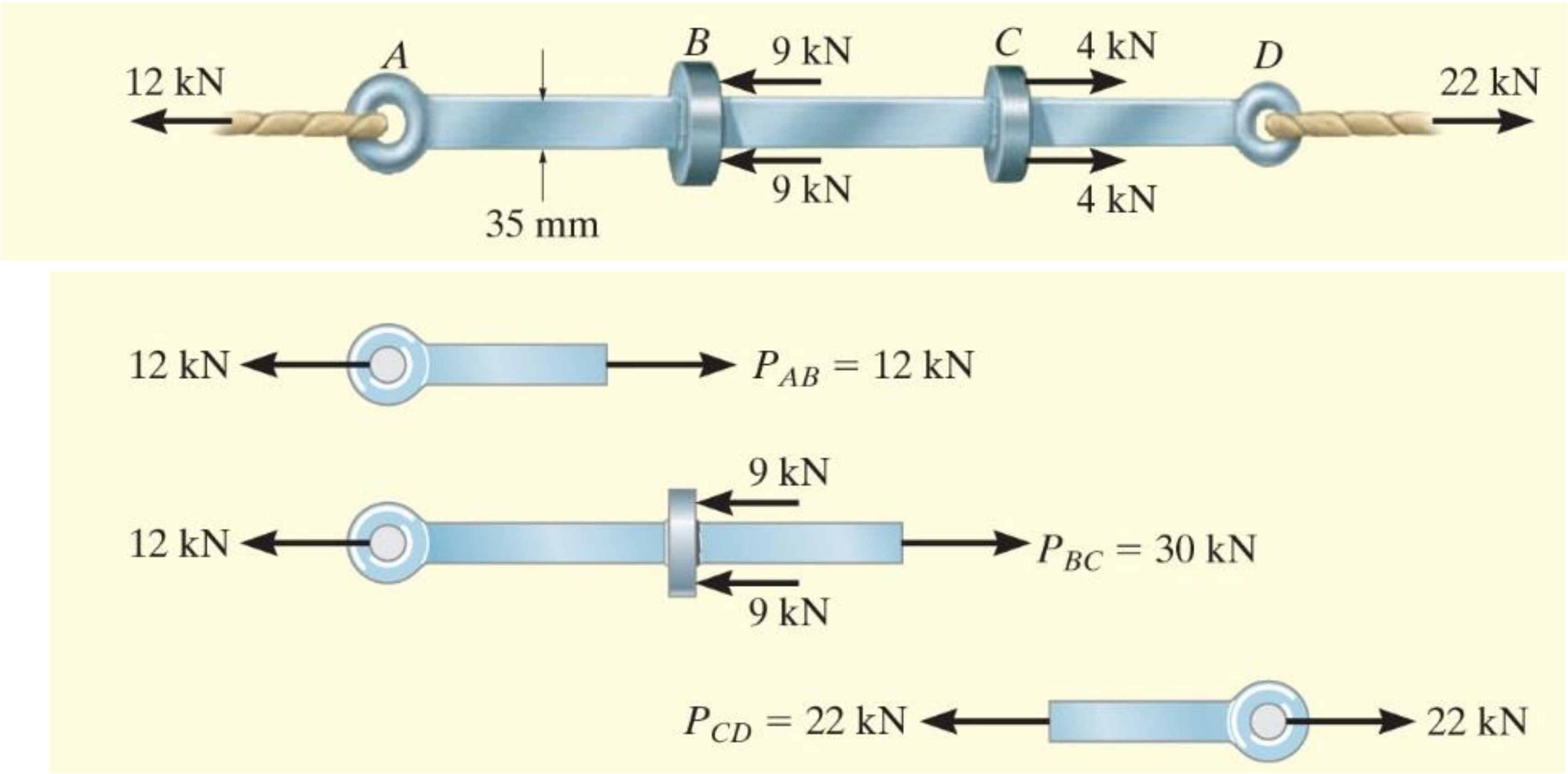
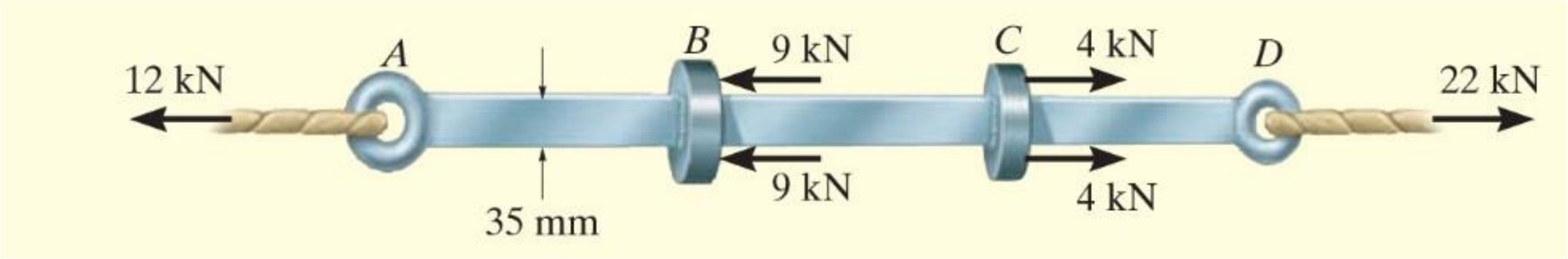
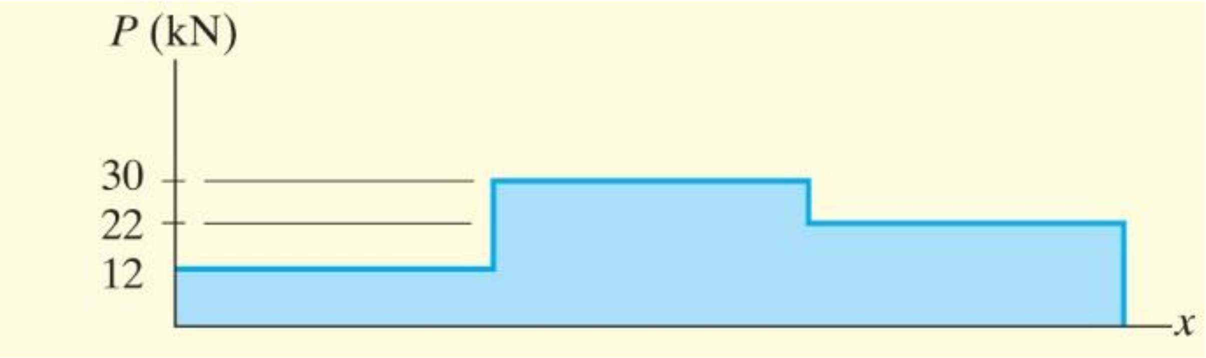
Example 1

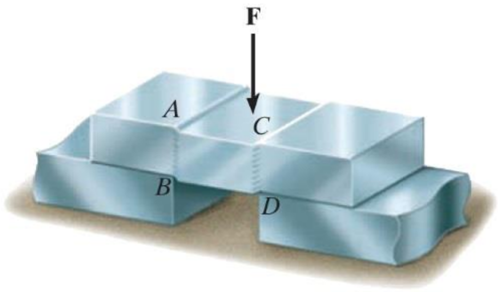

12 kN
A
B
9 kN
C
4 kN
D
22 kN
9 kN
4 kN
35 mm
12 kN
P_{AB} = 12 kN
9 kN
12 kN
P_{BC} = 30 kN
9 kN
P_{CD} = 22 kN
22 kN

Example 1

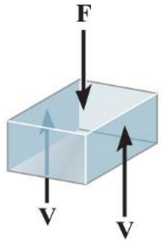




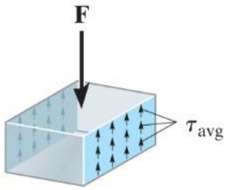
Average Shear Stress



(a)



(b)



(c)

Shear stress is the stress component that acts *in the plane* of the sectioned area.

To show how this stress can develop, consider the effect of applying a force **F** to a bar. If the supports are considered rigid, and **F** is large enough, it will cause the material of the bar to deform and fail along the planes identified by *AB* and *CD*.

A free-body diagram of the unsupported center segment of the bar, indicates that the shear force V=F/2 must be applied at each section to hold the segment in equilibrium. The ***average shear stress*** distributed over each sectioned area that develops this shear force is defined by

Average Shear Stress

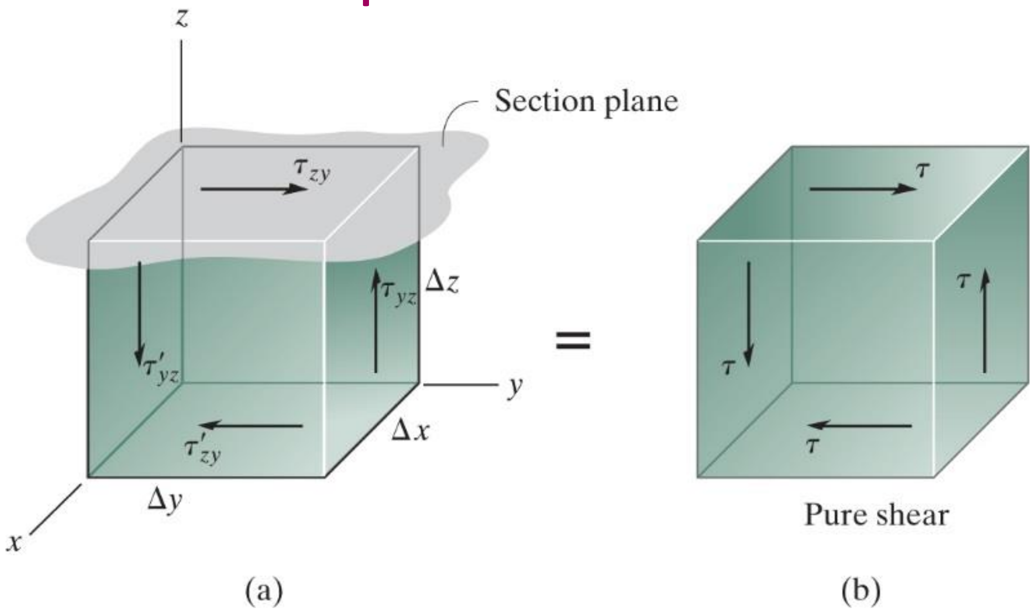
$$\tau_{avg} = \frac{V}{A}$$

Here

τ	average shear stress at the section, which is assumed to be the *same* at each point located on the section
V	internal resultant shear force on the section determined from the equations of equilibrium
A	area at the section

Average Shear Stress

Shear Stress Equilibrium.



volume element

$$\sum F_y = 0 \Rightarrow \tau_{zy}(\Delta x.\Delta y) - \tau'_{zy}(\Delta x.\Delta y) = 0 \Rightarrow \tau_{zy} = \tau'_{zy}$$

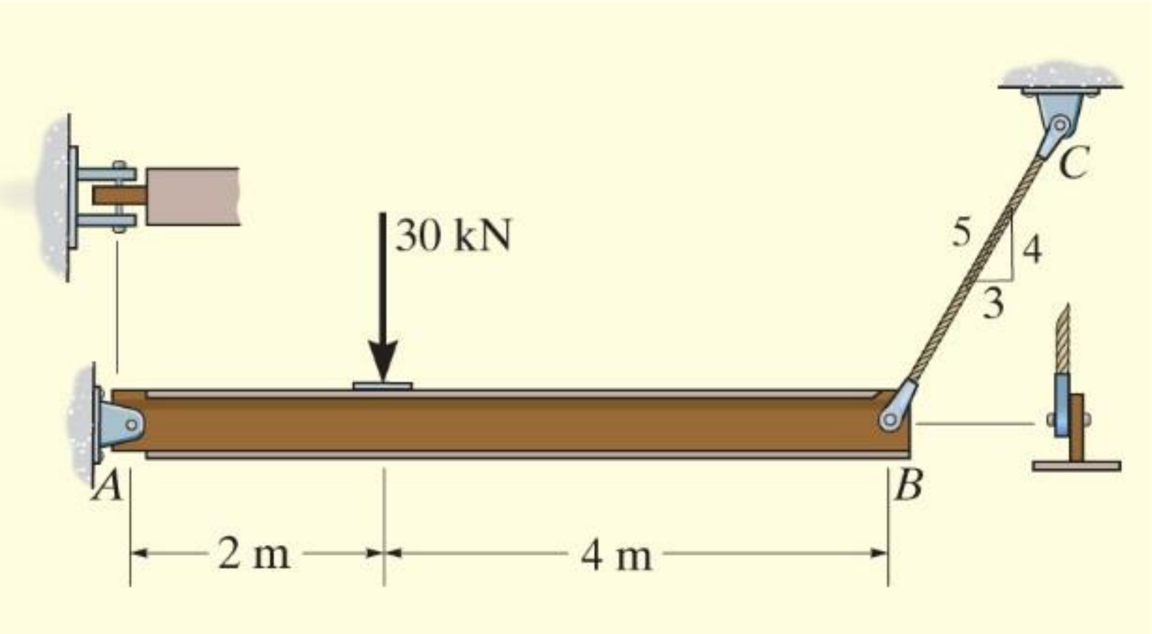
$$\sum M_x = 0 \Rightarrow -\tau_{zy}(\Delta x.\Delta y)\Delta z + \tau_{yz}(\Delta x.\Delta z)\Delta y = 0 \Rightarrow \tau_{zy} = \tau_{yz}$$

Average Shear Stress

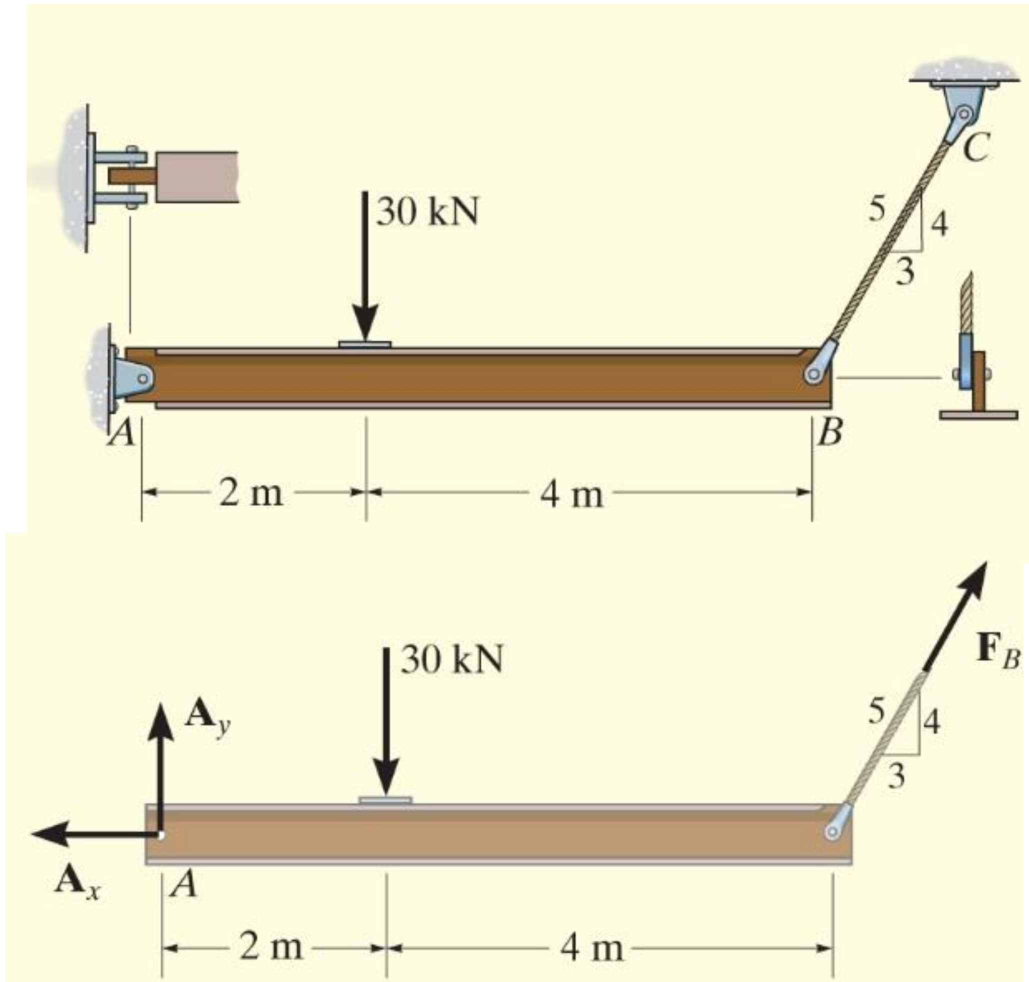
All four shear stresses must have equal magnitude and be directed either toward or away from each other at opposite edges of the element

Average Shear Stress

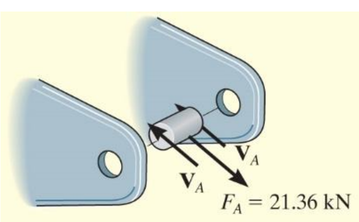
Determine the average shear stress in the 20-mm-diameter pin at *A* and the 30-mm-diameter pin at *B* that support the beam



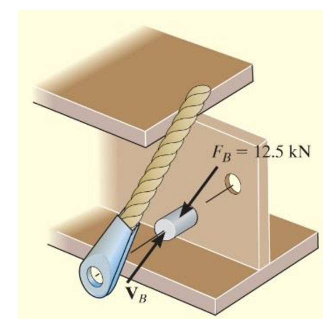
Example 2



30 kN
C
5
4
3
A
B
2 m
4 m
30 kN
F_B
A_y
A_x
A
5
4
3
2 m
4 m



V_A
V_A
F_A = 21.36 kN

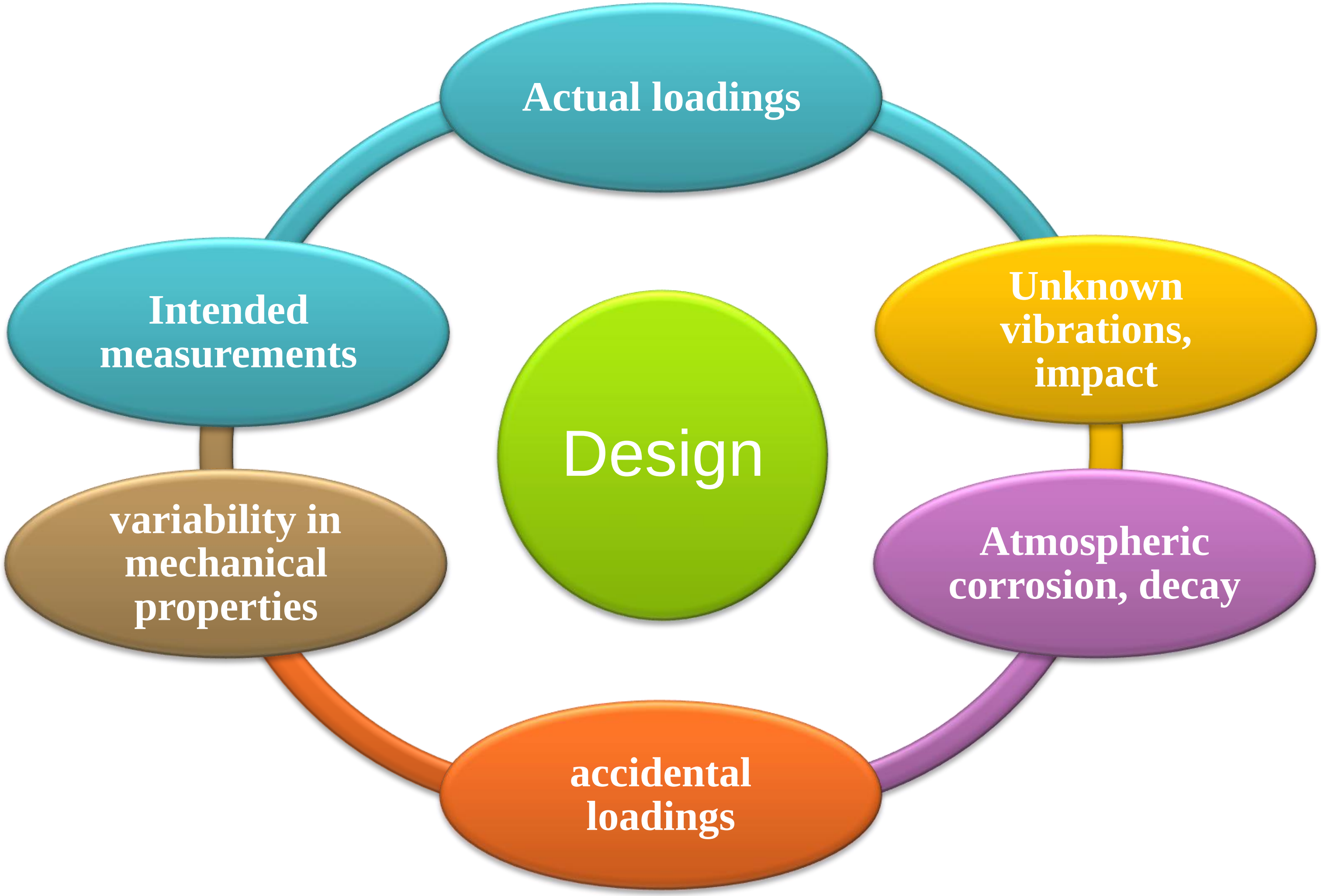


F_B = 12.5 kN
V_B

Design Loadings ↔ Actual Loadings

Are
Actual Loadings
similar to
Design Loadings

Design Loadings ↔ Actual Loadings



Allowable Stress

To properly *design* a structural member or mechanical element it is necessary to restrict the stress in the material to a level that will be safe.

To ensure this safety, it is therefore necessary to choose an allowable stress that restricts the applied load to one that is *less* than the load the member can fully support.

Allowable Stress

One method of specifying the allowable load for a member is to use a number called the factor of safety. The ***factor of safety*** (F.S.) is a ratio of the failure load F_{fail} to the allowable load F_{allow} Here is found from experimental testing of the material, and the factor of safety is selected based on experience so that the above mentioned uncertainties are accounted for when the member is used under similar conditions of loading and geometry. Stated mathematically,

$$F.S = \frac{F_{fail}}{F_{allow}}$$

Allowable Stress

If the load applied to the member is *linearly related* to the stress developed within the member, as in the case of using and then we can also express the factor of safety as a ratio of the failure stress σ_{fail} (or τ_{fail}) to the allowable stress σ_{allow} (or τ_{allow}); that is,

$$F.S = \frac{\sigma_{fail}}{\sigma_{allow}} \qquad F.S = \frac{\tau_{fail}}{\tau_{allow}}$$

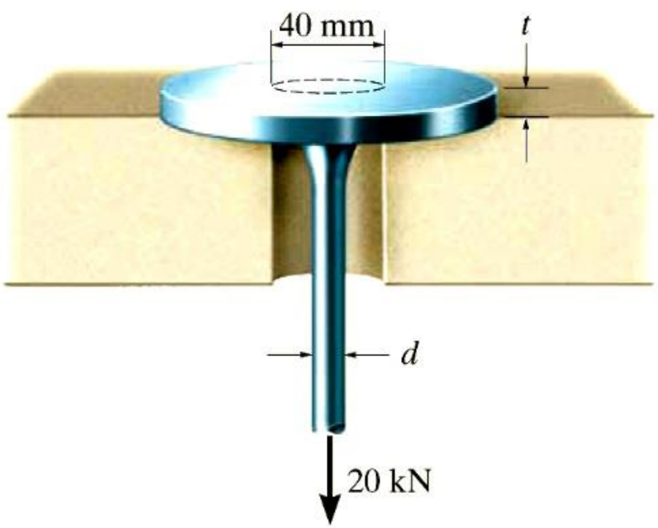
$$F.S > 1$$

Allowable Stress

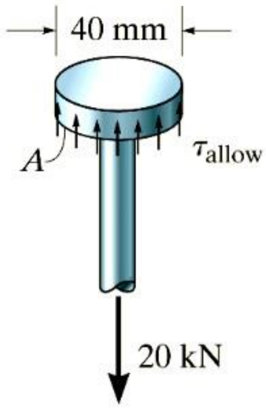
What is the relationship between the load, geometry and material) and factor of safety ????

Example 3

The suspender rod is supported at its end by a fixed-connected circular disk as shown in Fig. *a*. If the rod passes through a 40-mm-diameter hole, determine the minimum required diameter of the rod and the minimum thickness of the disk needed to support the 20-kN load. The allowable normal stress for the rod is $\sigma_{allow} = 60$ MPa, and the allowable shear stress for the disk is $\tau_{allow} = 35$ MPa.



(a)



(b)