Mechanics of Materials

Lecture 1

General Review

Mohamad Fathi GHANAMEH

Lecture Topics

- ✓ Course overview
- ✓ Review some of the important principles of statics and show how they are used to determine the internal resultant loadings in a body.
- ✓ Concepts of normal and shear stress will be introduced, and specific applications of the analysis and design of members subjected to an axial load or direct shear will be discussed.

Course Organization

Instructor
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Lectures

Wednesday 08:30 - 10:30 AM

Tutorials

Gr A: Wednesday	16:00 - 18:00 PM
Gr B : Thursday	10:30 - 12:30 AM
Gr C: Wednesday	14:00 - 16:00 PM
Gr D: Thursday	08:30 - 10:30 AM

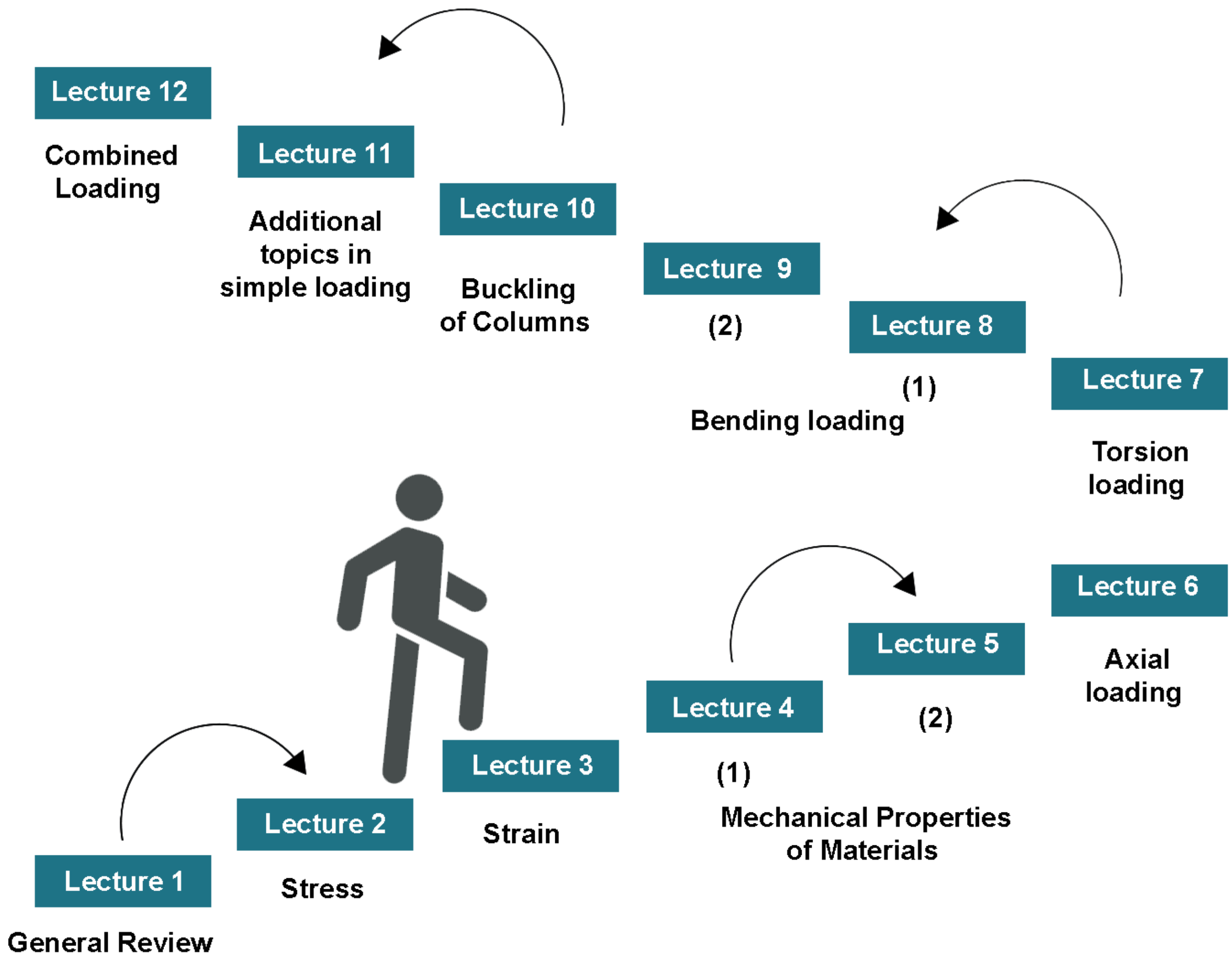
Course Organization

Course Documentation and Support
http://ghanameh.tarkiah.com/index.php/courses/mechanics-of-materials

Office Hours
Monday 09:30 - 11:30 AM

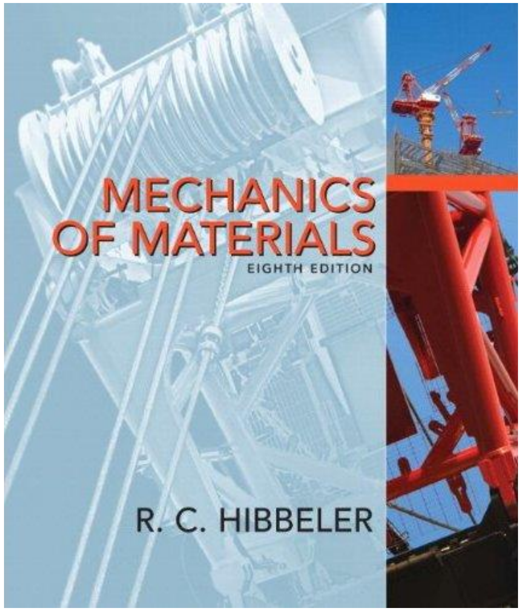
Evaluation
Quizzes, Mid-term, Final exam, Assignements and Mini Project

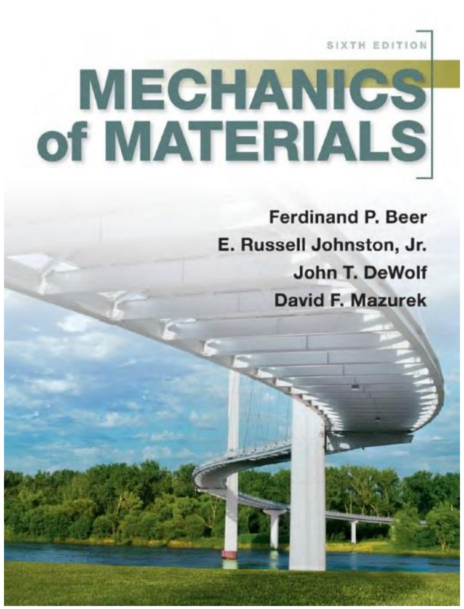
Course Outline

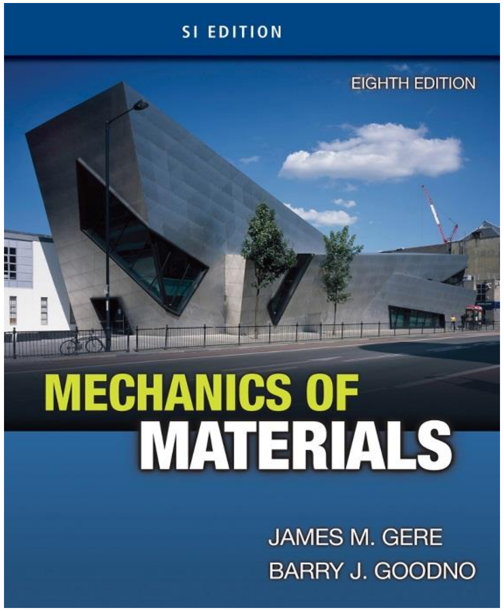


Textbook & References

- ✓ R. C. Hibbeler, *Mechanics Of Materials*, 8th Edition, Pearson Prentice Hall, 2011.
- ✓ James M. Gere, Barry J. Goodno, *Mechanics Of Materials - SI Edition*, 8th Edition, Cengage Learning, 2013.
- ✓ Ferdinand P. Beer, E. Russell Johnston Jr., John T. Dewolf, David F. Mazurek, *Mechanics Of Materials*, 6th Edition, McGrawHill, 2012.

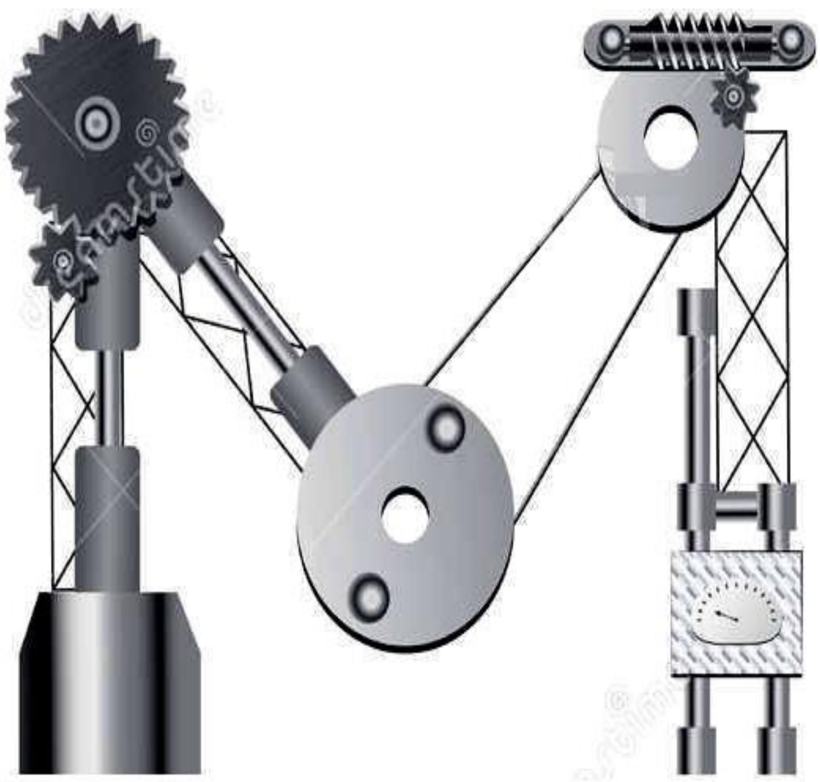






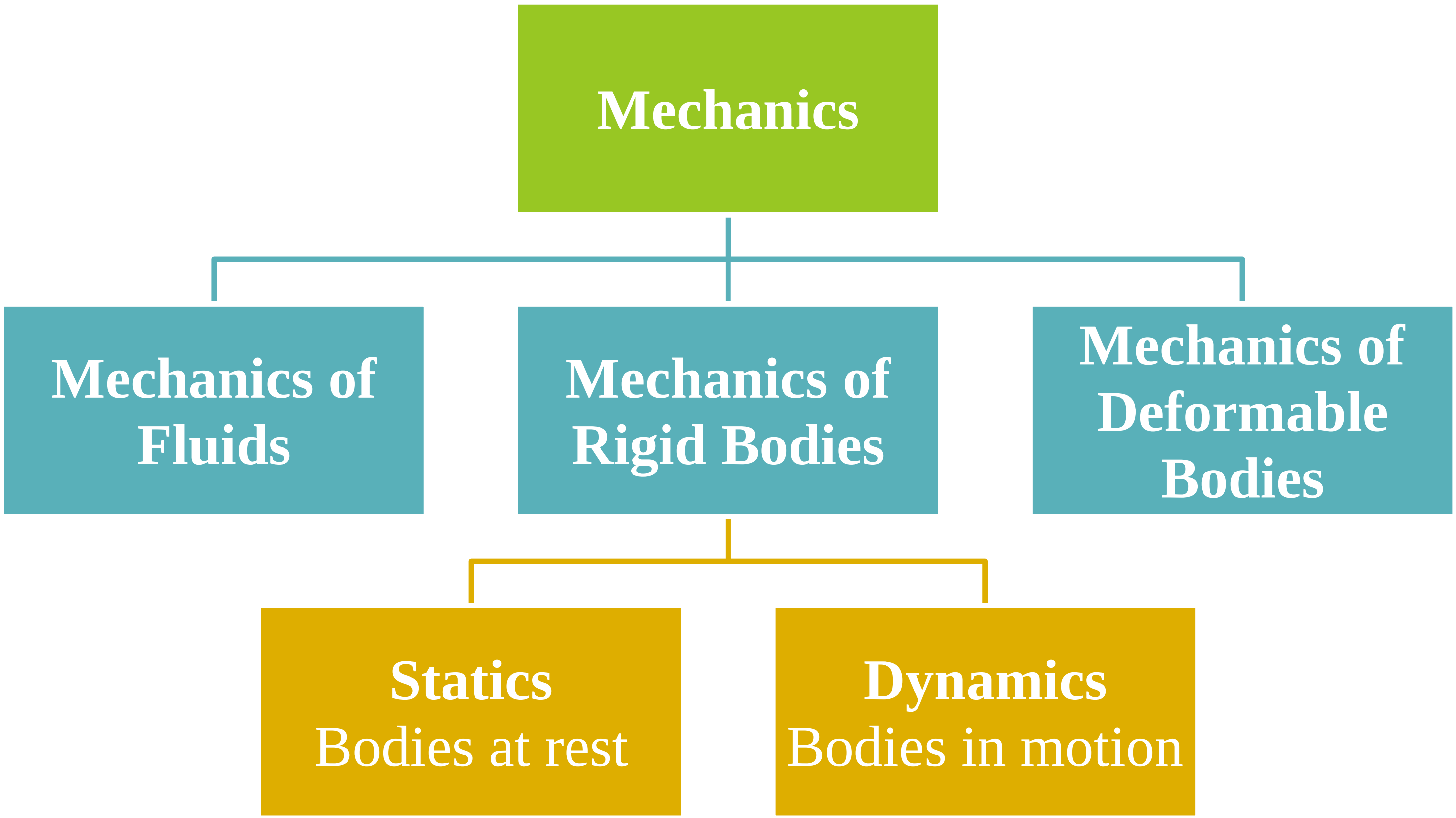
What is the Mechanics?

Mechanics can be defined as the science which describes the condition of rest or motion of bodies under the action of forces.

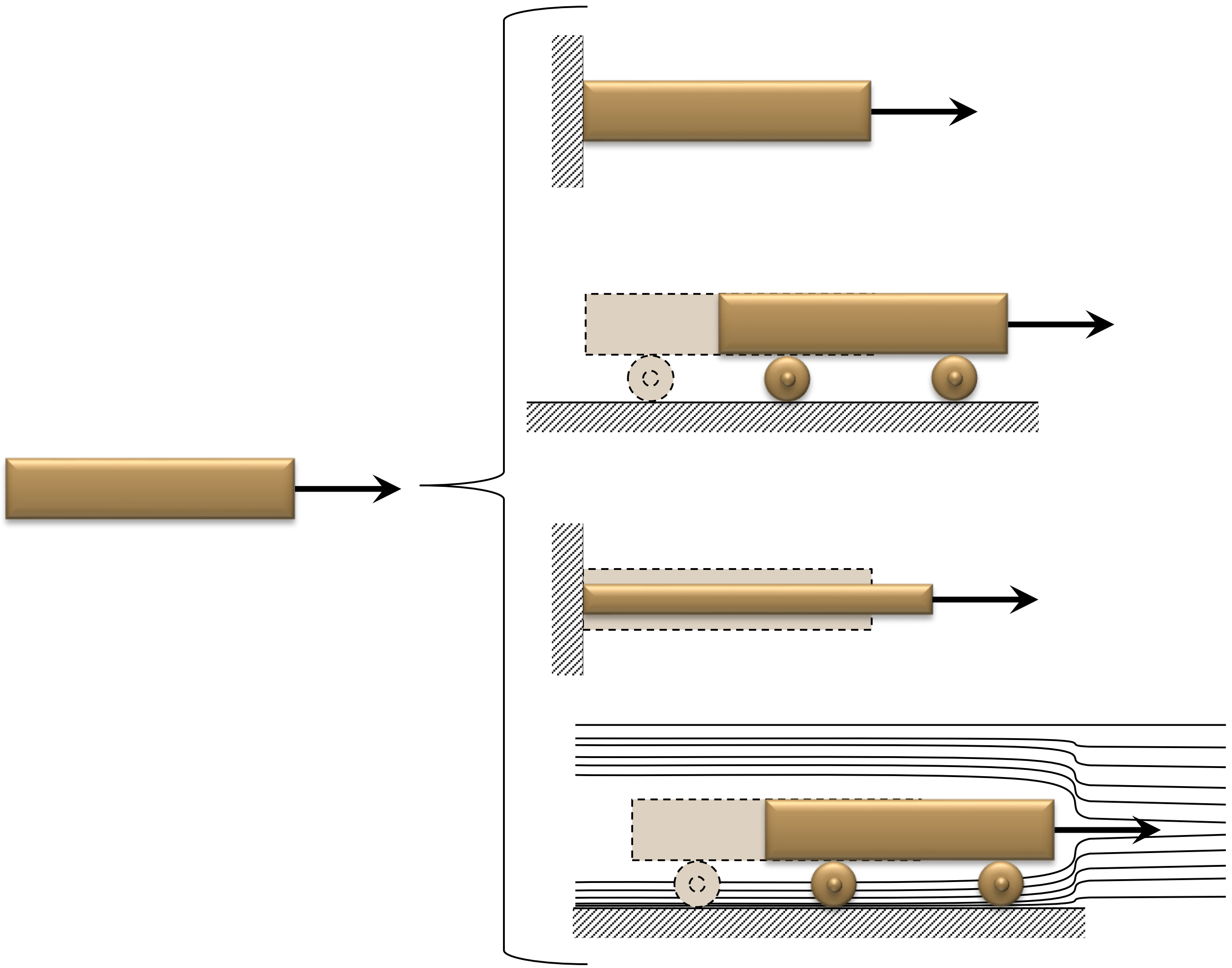


ECHANICS

What is the Mechanics?



What is the Mechanics?



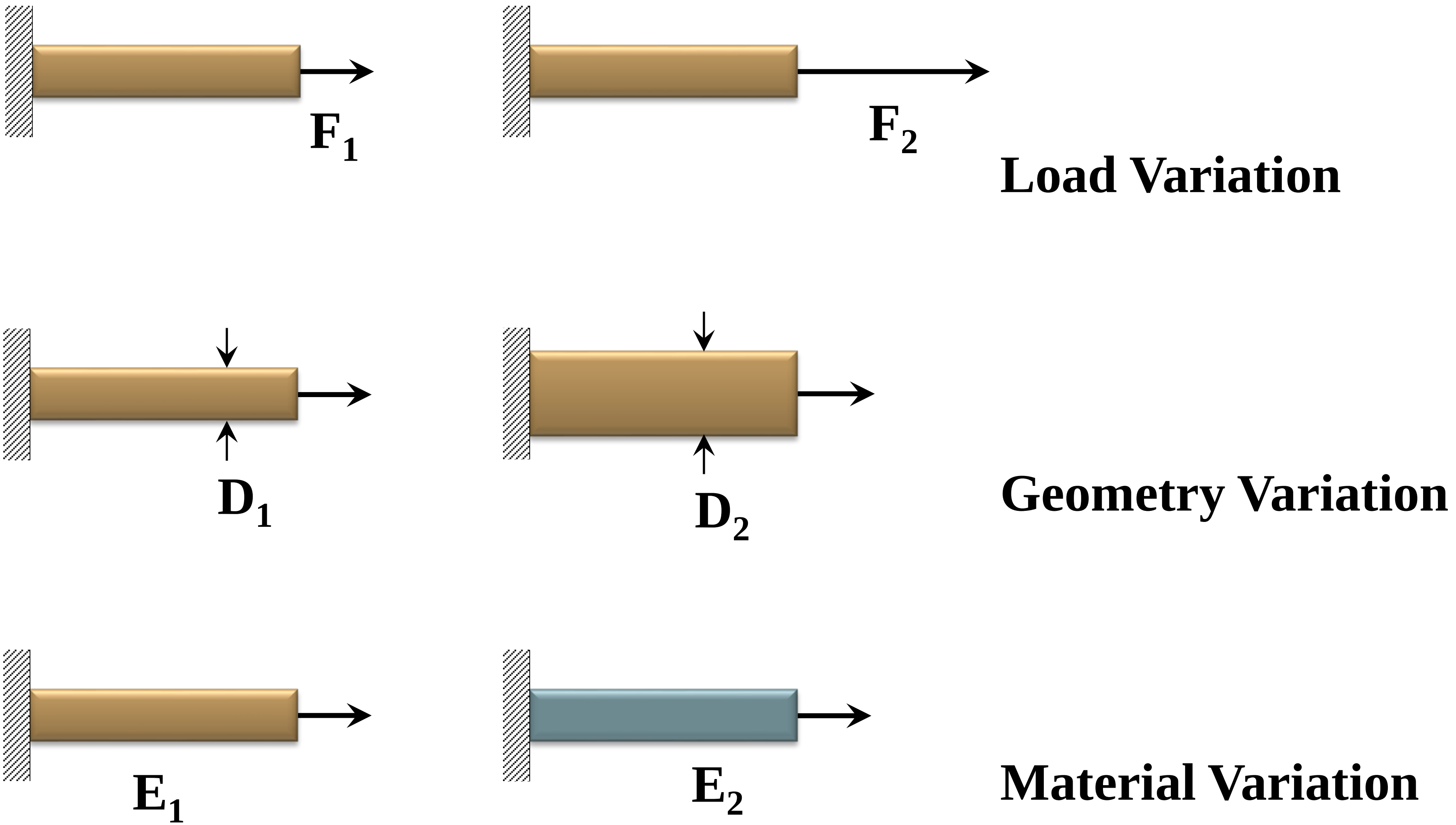
Statics
Bodies at rest

Dynamics
Bodies in motion

Mechanics of Deformable Bodies

Mechanics of Fluids

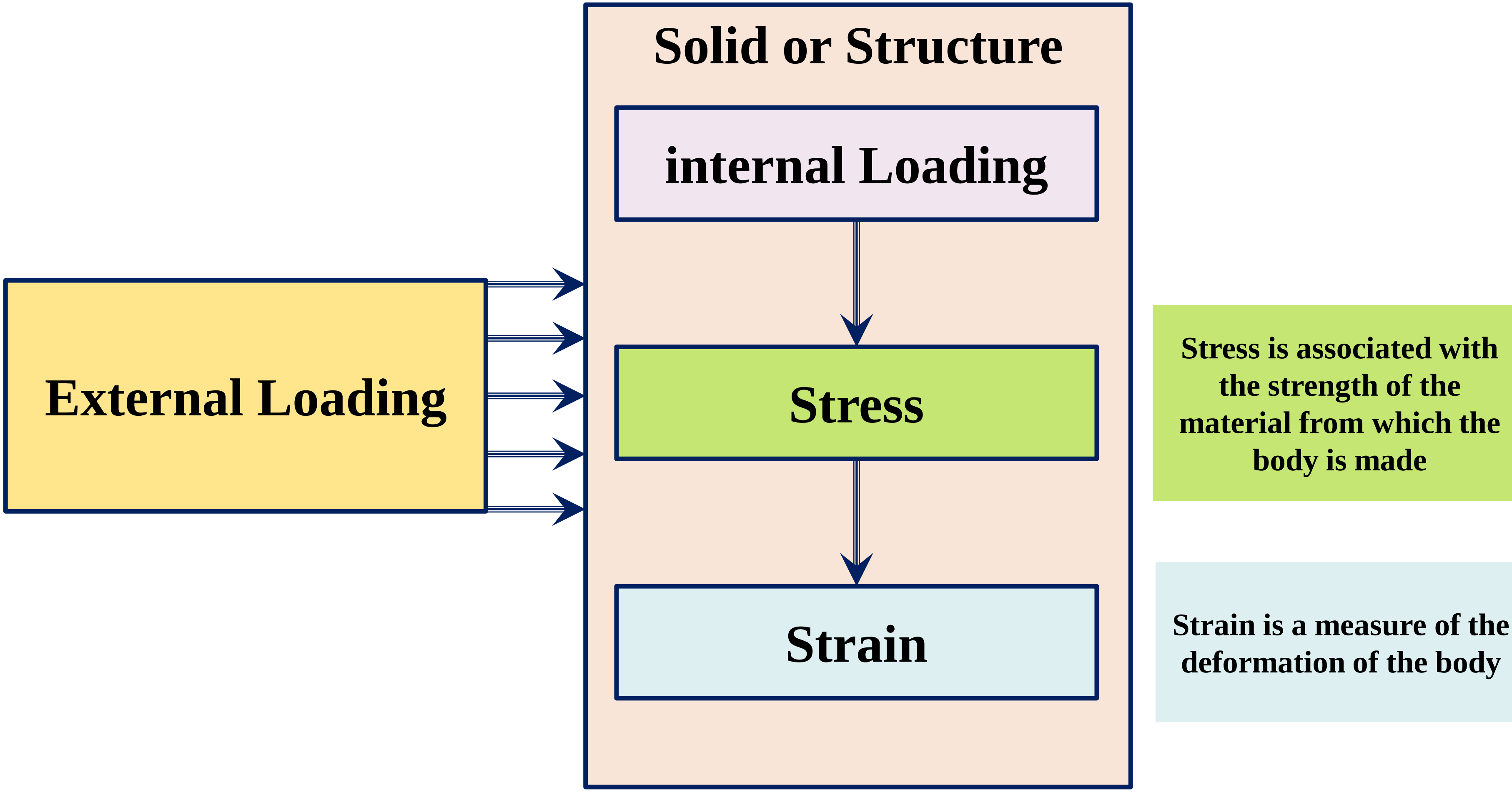
Mechanics of materials



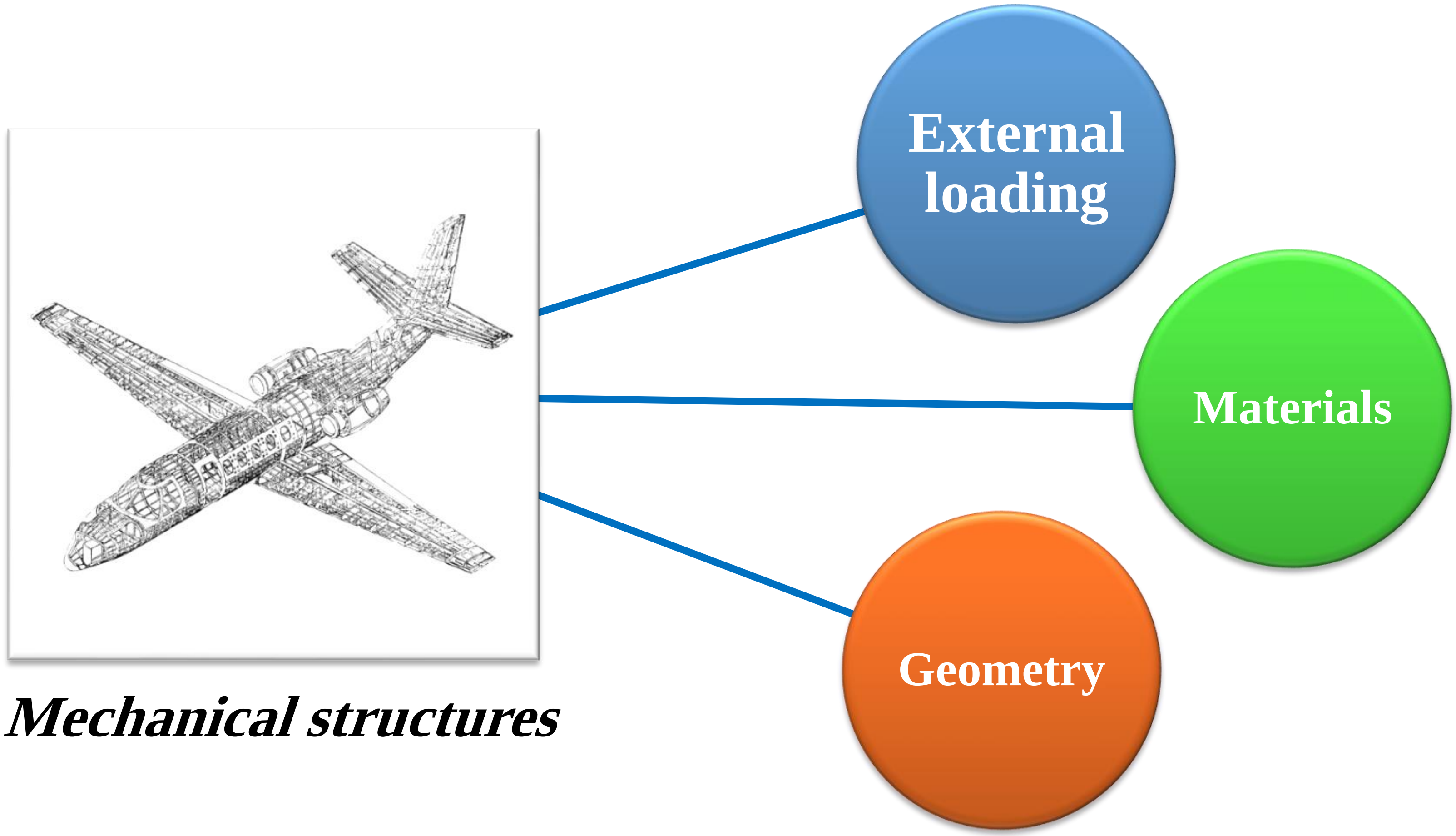
Mechanics of materials

- Studies the internal effects of stress and strain in a solid body that is subjected to an external loading.
- study of the body's stability when a body is subjected to defined loading

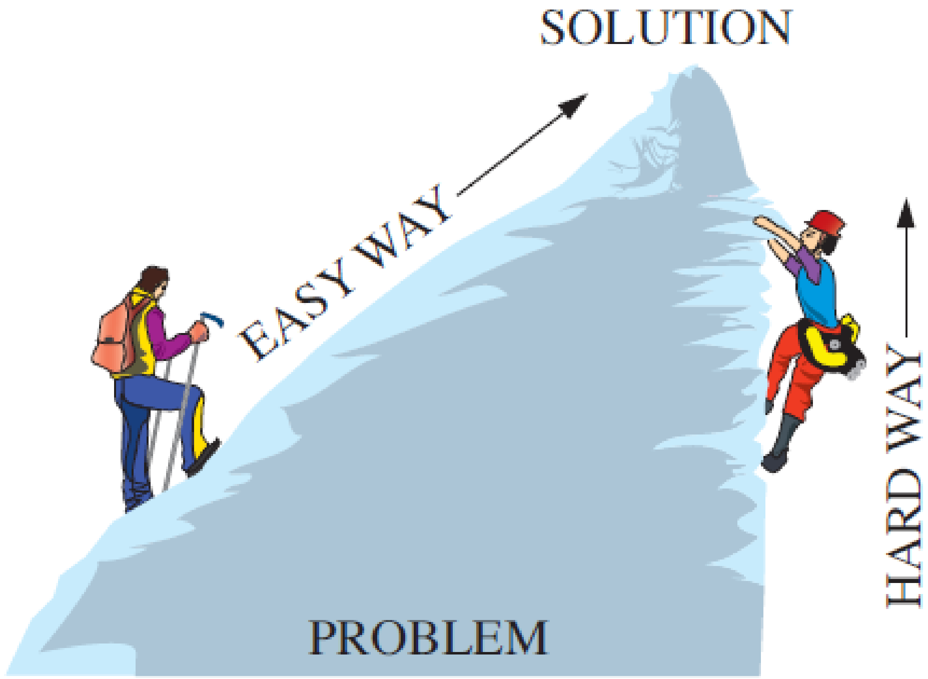
Mechanics of materials



Mechanics of materials



Problem-solving



A step-by-step approach can greatly simplify problem solving.

Problem-solving Procedures

For a given problem, three steps are used to solve the problems correctly:

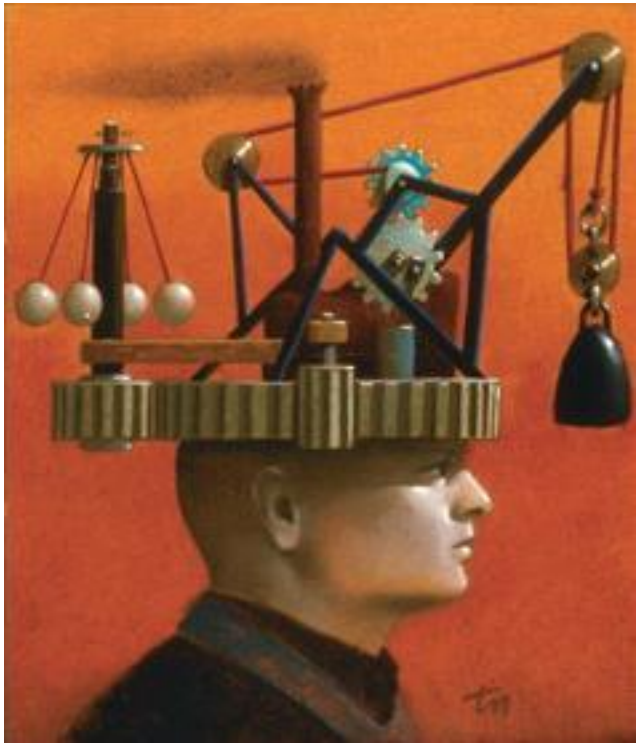
Plan the solution

↓

Carry out the solution

↓

Review the solution



Problem-solving Procedures

For a given problem, three steps are used to solve the problems correctly:

A. Plan the solution: plan strategy and steps in solving the problems
B. Carry out the solution: find the proper tools to solve the problems using equilibrium equations, geometry of deformation, and material type.
C. Review the solution: Does the dimension make sense? Are the quantities in a reasonable manner including sign and magnitude? Does the solution violate the assumption you assumed before solving the problem?

External Loading

External Loads

Surface Forces

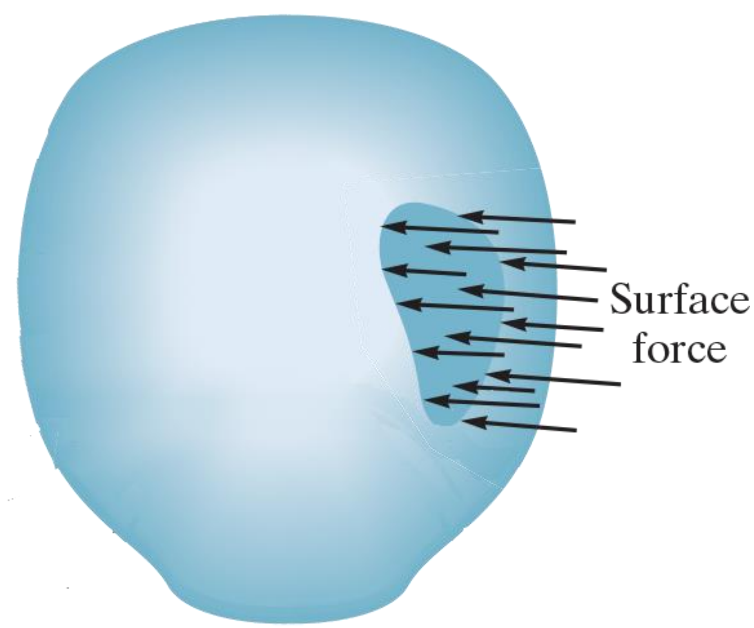
caused by the direct contact of one body with the surface of another

Body Forces

developed when one body exerts a force on another body without direct physical contact between the bodies

Surface forces

- Are caused by the direct contact of one body with the surface of another.
- Are distributed over the area of contact between the bodies.
- The loading is measured as having an intensity of force/surface over the area and is represented graphically by a series of arrows over the area.



- The resultant force : equivalent to $F = w(\text{s}).\text{A}$ and acts centroid C or geometric center

Surface forces

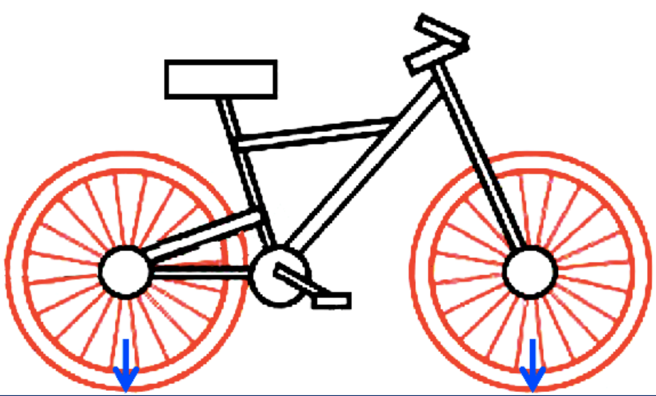
Surface Forces

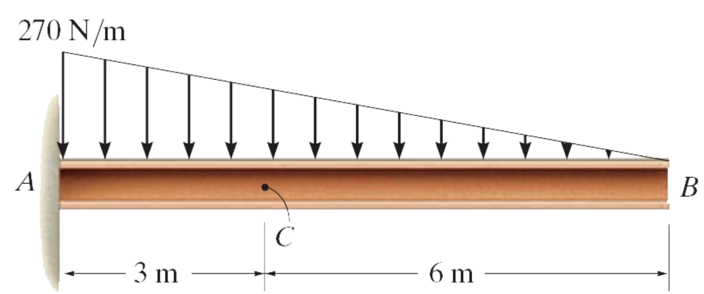
Concentrated

area is small in comparison with the total surface area of the body

Linear Distributed

the surface loading is applied along a narrow strip of area

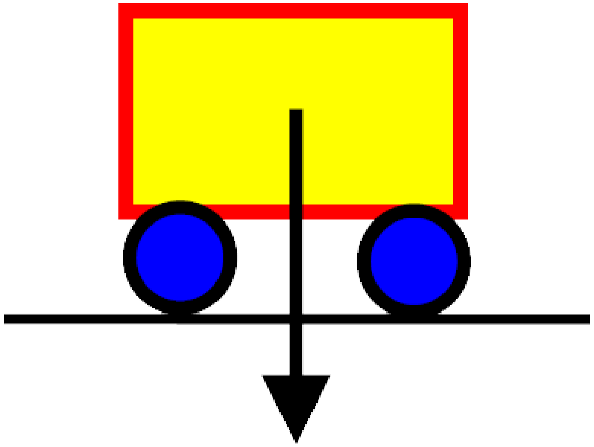




Body Force

Is developed when one body exerts a force on another body without direct physical contact between the bodies.

Examples include the effects caused by the earth's gravitation or its electromagnetic field. Although body forces affect each of the particles composing the body, these forces are normally represented by a single concentrated force acting on the body. In the case of gravitation, this force is called the ***weight*** of the body and acts through the body's center of gravity.



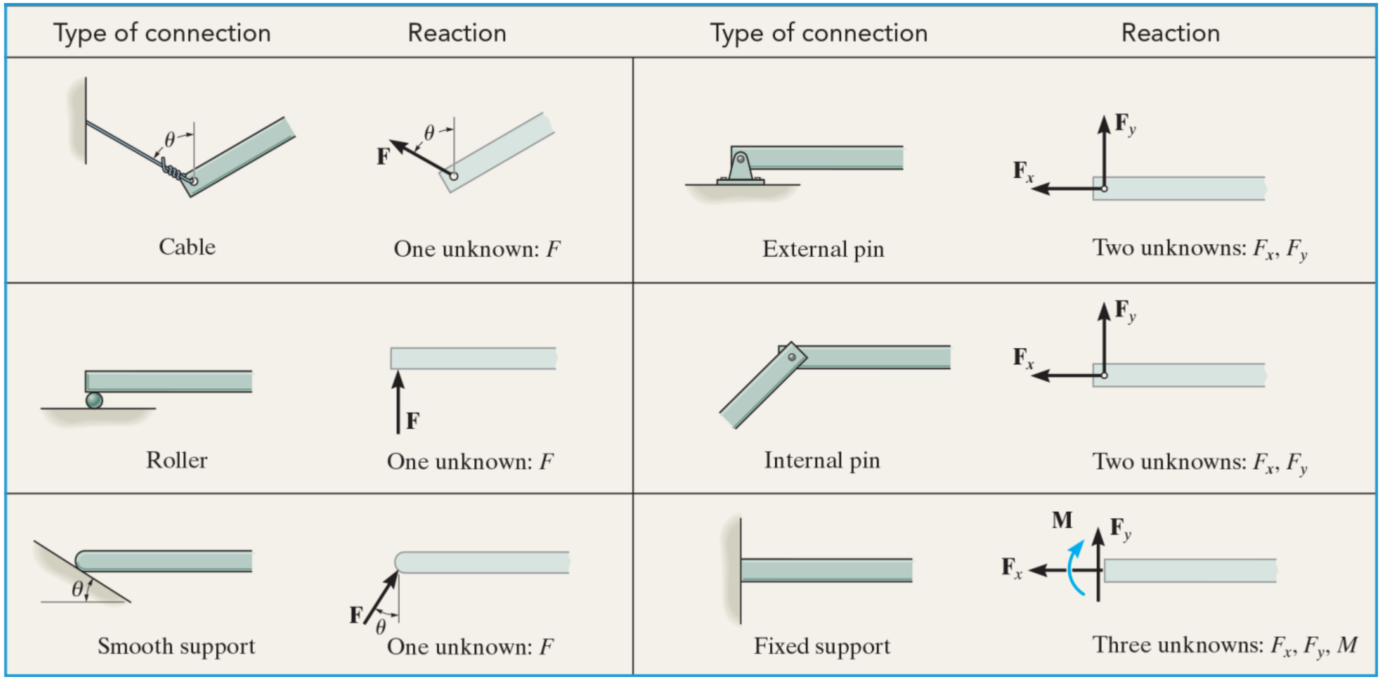
$$W = m.g$$

Support Reactions

The surface forces that develop at the supports or points of contact between bodies are called ***reactions***.
If the support prevents translation in a given direction, then a force must be developed on the member in that direction. Likewise, if rotation is prevented, a couple moment must be exerted on the member

Support Reactions

The supports most commonly encountered for 2D problems, i.e., bodies subjected to coplanar force systems:



Note carefully the symbol used to represent each support and the type of reactions it exerts on its contacting member

Equilibrium of a Deformable Body

Equilibrium of a body requires both:

a ***balance of forces***, to prevent the body from translating or having accelerated motion along a straight or curved path, and

a ***balance of moments***, to prevent the body from rotating. These conditions can be expressed mathematically by two vector equations

$$\sum F = 0$$

$$\sum M_o = 0$$

Equilibrium of a Deformable Body

The force and moment vectors can be resolved into components along each coordinate axis and the two equations can be written in scalar form as six equations, namely

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$$

$$\sum M_x = 0, \sum M_y = 0, \sum M_z = 0$$

For planar system : for example in the plane xoy

$$\sum F_x = 0, \sum F_y = 0$$

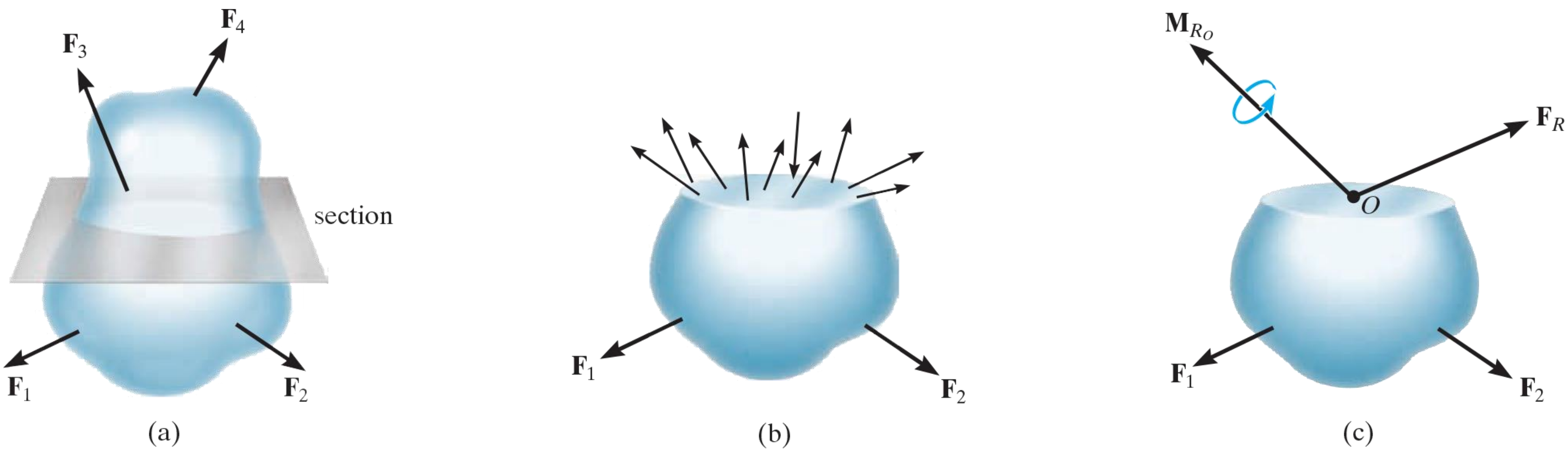
$$\sum M_o = 0$$

Equilibrium of a Deformable Body

Successful application of the equations of equilibrium requires complete specification of all the known and unknown forces that act *on* the body, and so ***the best way to account for all these forces is to draw the body's free-body diagram***.

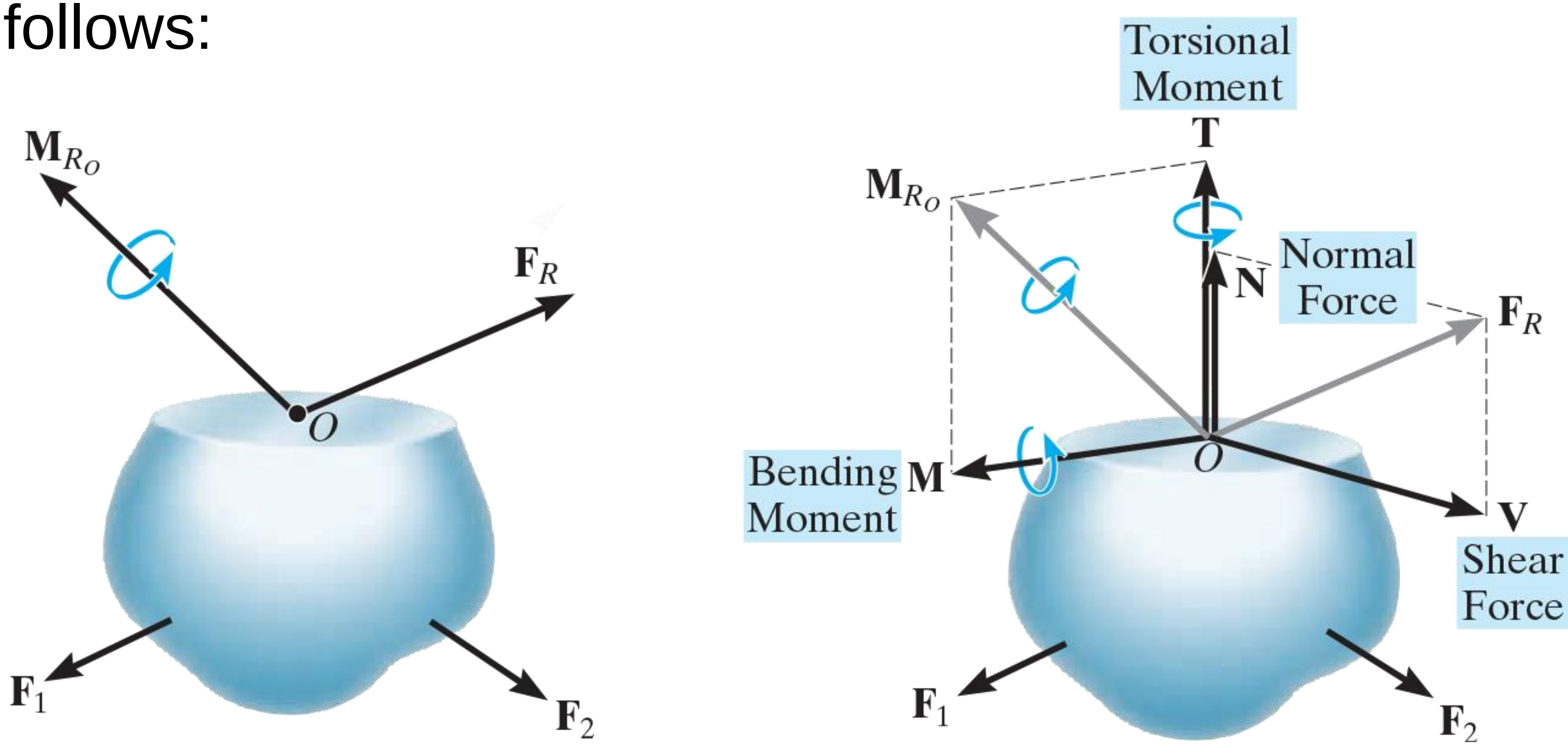
Internal Resultant Loadings

In mechanics, statics is primarily used to determine the resultant loadings that act within a body.



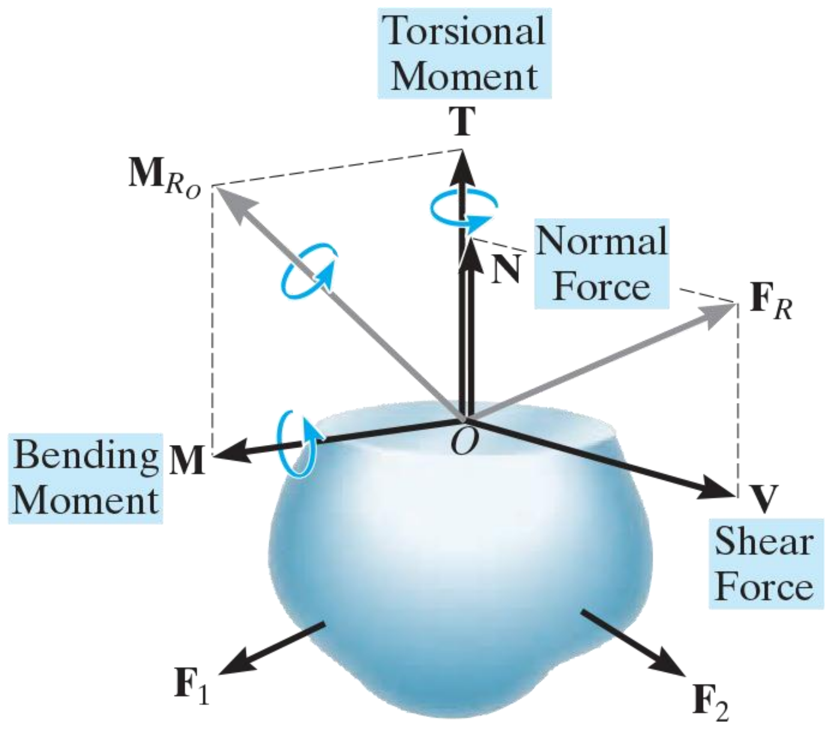
Three Dimensions

The components and acting of both normal and perpendicular to the sectioned area must be considered, Four different types of resultant loadings can then be defined as follows:



Three Dimensions

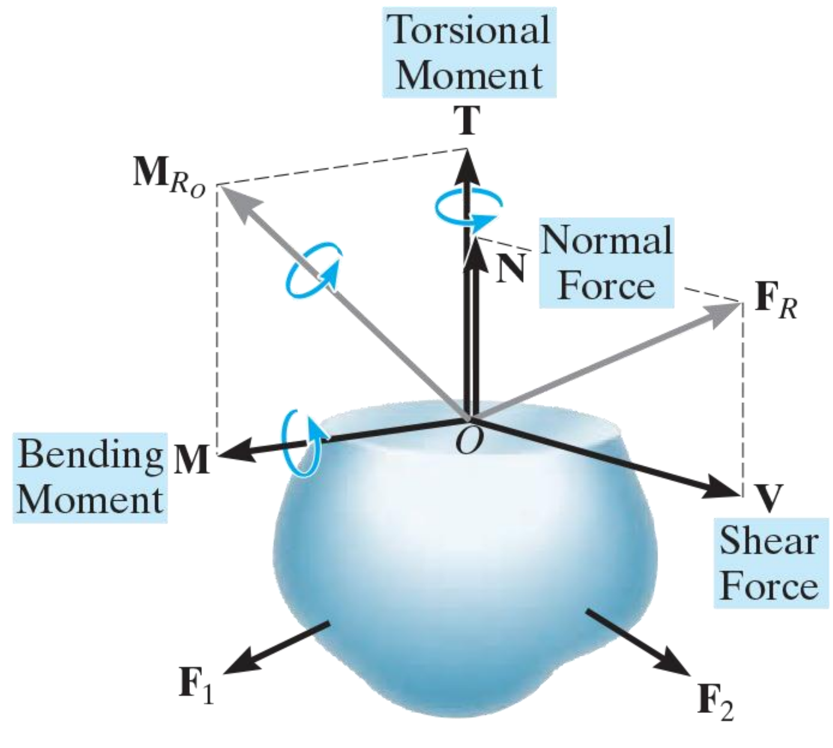
Normal force, N. This force acts perpendicular to the area. It is developed whenever the external loads tend to push or pull on the two segments of the body.



(d)

Three Dimensions

Shear force, V. The shear force lies in the plane of the area and it is developed when the external loads tend to cause the two segments of the body to slide over one another.



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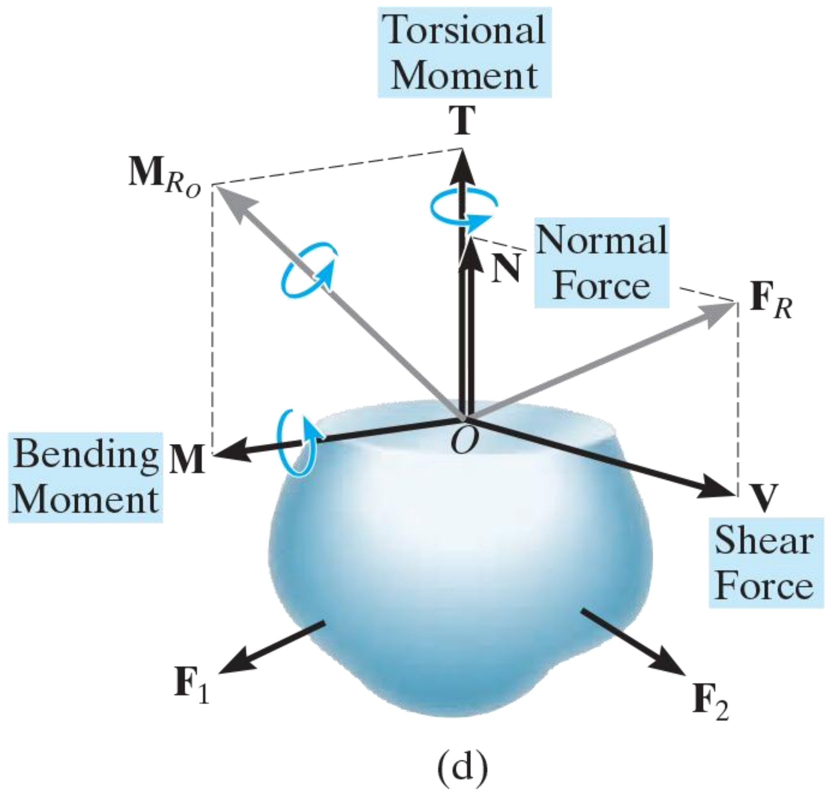
Three Dimensions

Torsional moment or torque, T. This effect is developed when the external loads tend to twist one segment of the body with respect to the other about an axis perpendicular to the area.



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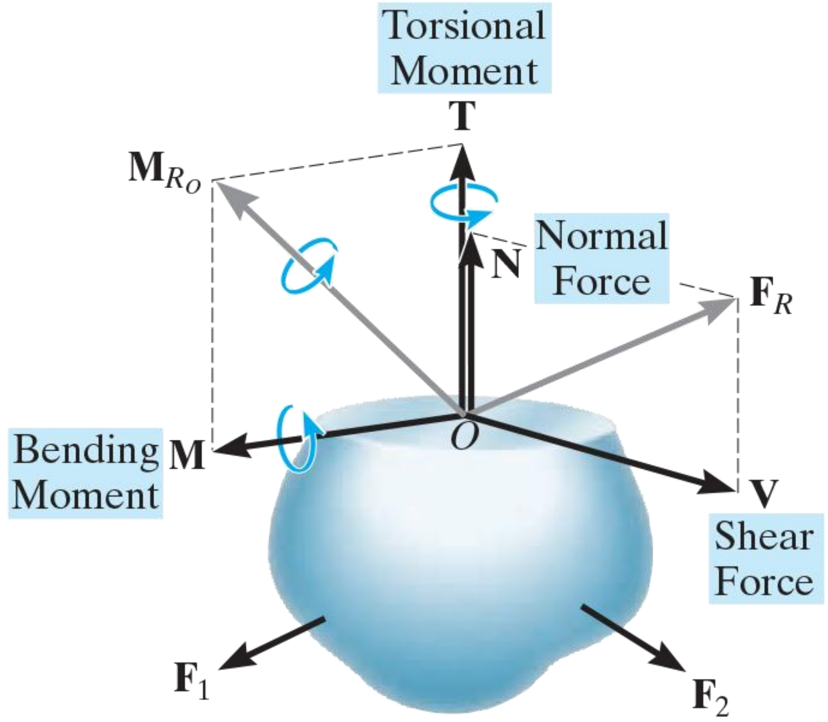
Three Dimensions

Bending moment, M. The bending moment is caused by the external loads that tend to bend the body about an axis lying within the plane of the area.



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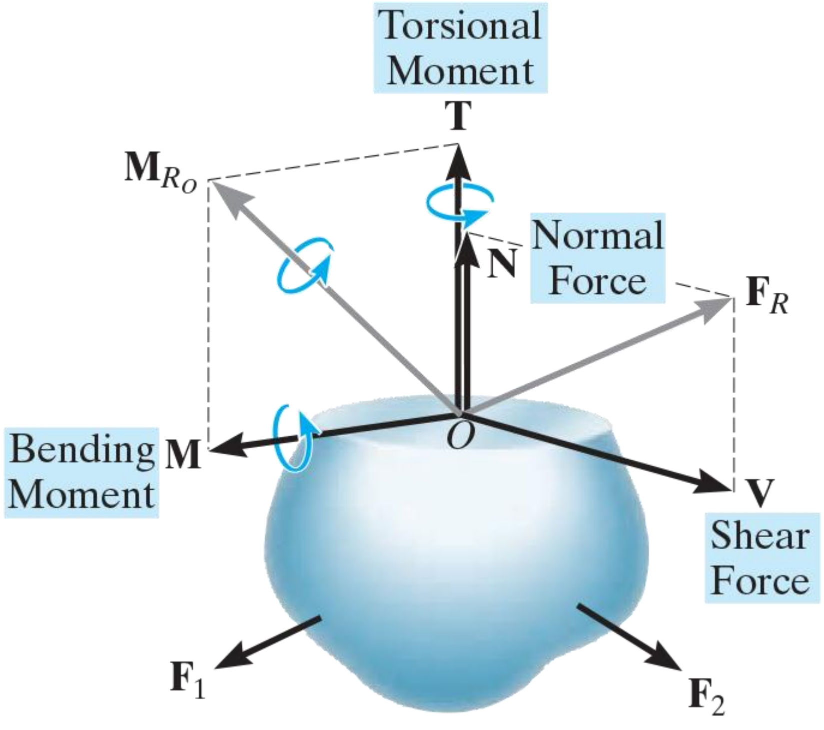
Three Dimensions

Note that graphical representation of a moment or torque is shown in three dimensions as a vector with an associated curl. By the *right hand rule*, the thumb gives the arrowhead sense of this vector and the fingers or curl indicate the tendency for rotation (twisting or bending).



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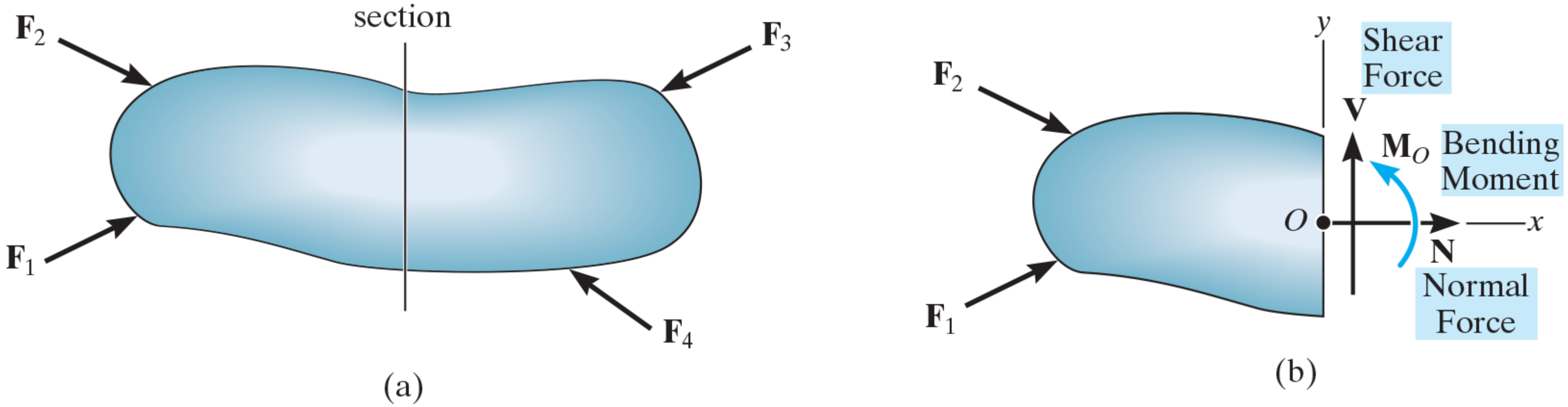
Coplanar Loadings.



When The body is subjected to a *coplanar system of forces* Only normal-force, shear-force, and bending-moment components will exist at the section.

Procedure for Analysis

The resultant *internal* loadings at a point located on the section of a body can be obtained using the method of sections. This requires the following 3 steps:

1. Support Reactions.
2. Free-Body Diagram.
3. Equations of Equilibrium.

1- Support Reactions

- ✓ Decide which segment of the body is to be considered.
- ✓ If the segment has a support or connection to another body, then *before* the body is sectioned, it will be necessary to determine the reactions acting on the chosen segment.
- ✓ To do this draw the free-body diagram of the *entire body* and then apply the necessary equations of equilibrium to obtain these reactions.

2- Free-Body Diagram

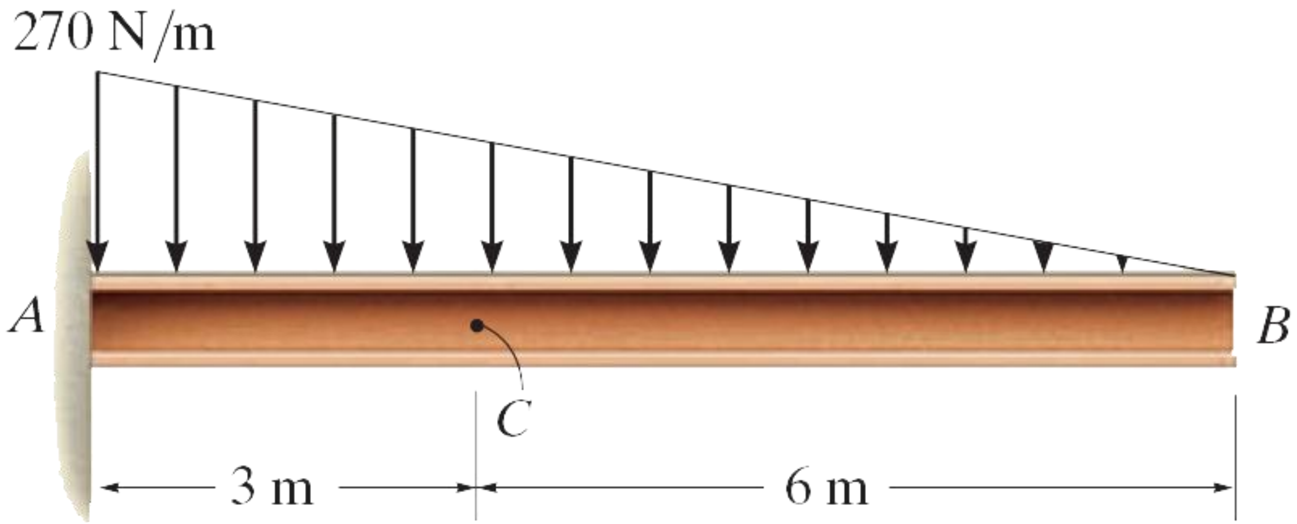
- ✓ Keep all external distributed loadings, couple moments, torques, and forces in their *exact locations*, before passing an imaginary section through the body at the point where the resultant internal loadings are to be determined.
- ✓ Draw a free-body diagram of one of the "cut" segments and indicate the unknown resultants **N**, **V**, **M**, and **T** at the section. These resultants are normally placed at the point representing the geometric center or *centroid* of the sectioned area.
- ✓ If the member is subjected to a *coplanar* system of forces, only **N**, **V**, and **M** act at the centroid.
- ✓ Establish the *x*, *y*, *z* coordinate axes with origin at the centroid and show the resultant internal loadings acting along the axes.

3- Equations of Equilibrium

- ✓ Moments should be summed at the section, about each of the coordinate axes where the resultants act. Doing this eliminates the unknown forces **N** and **V** and allows a direct solution for **M** (and **T**).
- ✓ If the solution of the equilibrium equations yields a negative value for a resultant, the assumed *directional sense* of the resultant is *opposite* to that shown on the free-body diagram.

Example 1

Determine the resultant internal loadings acting on the cross section at *C* of the cantilevered beam shown



Example 1

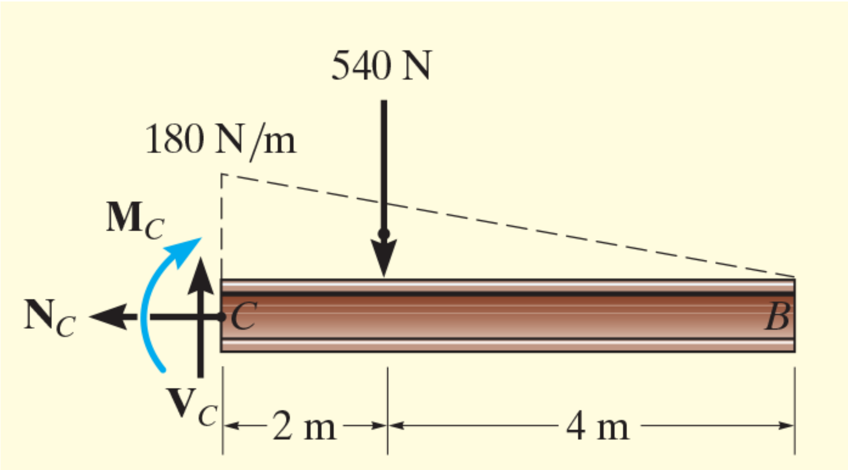
SOLUTION

1- Support Reactions:

The support reactions at *A* do not have to be determined if segment *CB* is considered.

2- Free-Body Diagram.

The free-body diagram of segment *CB* is



Example 1

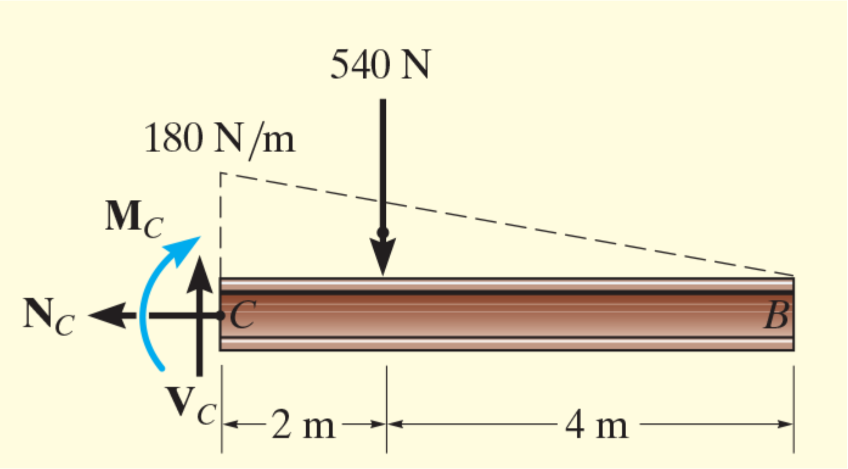
It is important to keep the distributed loading on the segment until *after* the section is made.

Only then should this loading be replaced by a single resultant force.

Notice that the intensity of the distributed loading at *C* is found by proportion.

$$\frac{W_1(s)}{L_1} = \frac{W_2(s)}{L_2} \Rightarrow \frac{270}{9} = \frac{W_c(s)}{6} \Rightarrow$$

$$W_c(s) = 6 \times 30 = 180 \quad N/m$$



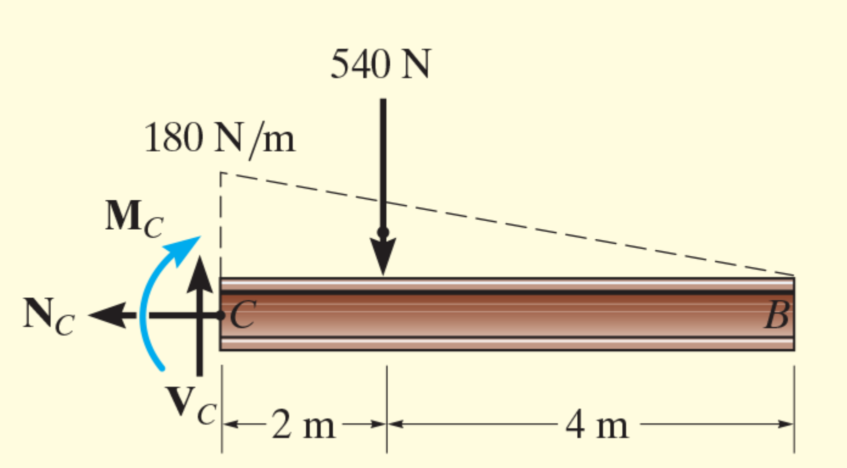
Example 1

The magnitude of the resultant of the distributed load is equal to the area under the loading curve (triangle) and acts through the centroid of this area.

$$F = \frac{W_c(s) \times CB}{2} = \frac{180 \times 6}{2} = 540 \quad N$$

Acts at : ⅓ of CB

$$\Rightarrow \frac{1}{3} \times 6 = 2 \quad m$$



Example 1

3- Equations of Equilibrium.

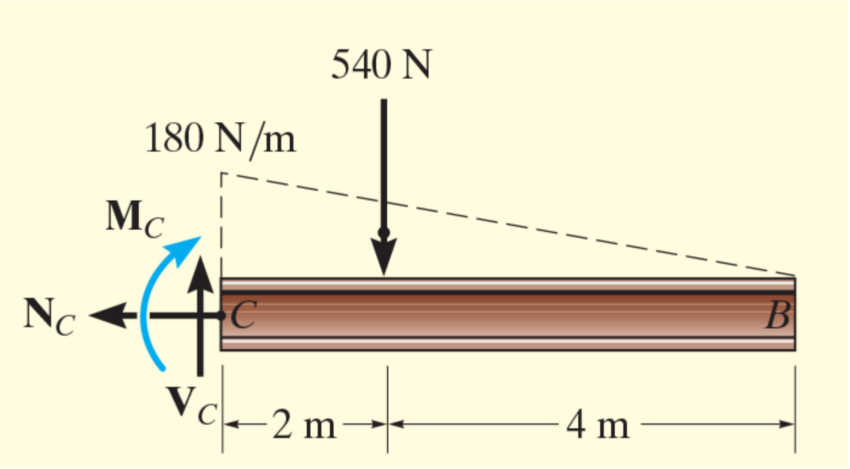
Applying the equations of equilibrium

$$\rightarrow \sum F_x = 0 \Rightarrow -N_c = 0 \Rightarrow N_c = 0 \quad N$$

$$\uparrow \sum F_y = 0 \Rightarrow V_c - 540N = 0 \Rightarrow V_c = 540 \quad N$$

$$\curvearrowleft \sum M_c = 0 \Rightarrow -M_c - 540N \times 2m = 0 \Rightarrow M_c = -1080 \quad N.m$$

The negative sign indicates that acts in the opposite direction to that shown on the free-body diagram.

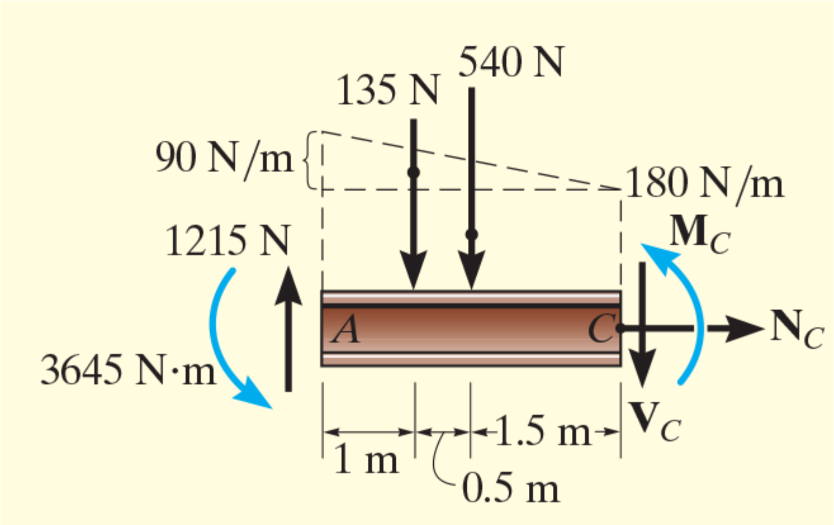


Example 1

The magnitude of the resultant of the distributed load is equal to the area under the loading curve (triangle + rectangle) and acts through the centroid of this area.

$$F_1 = \frac{W_{a1}(s) \times AC}{2} = \frac{90 \times 3}{2} = 135 \quad N \text{ acts at ⅓ of AC } \Rightarrow \frac{1}{3} \times 3 = 1 \quad m$$

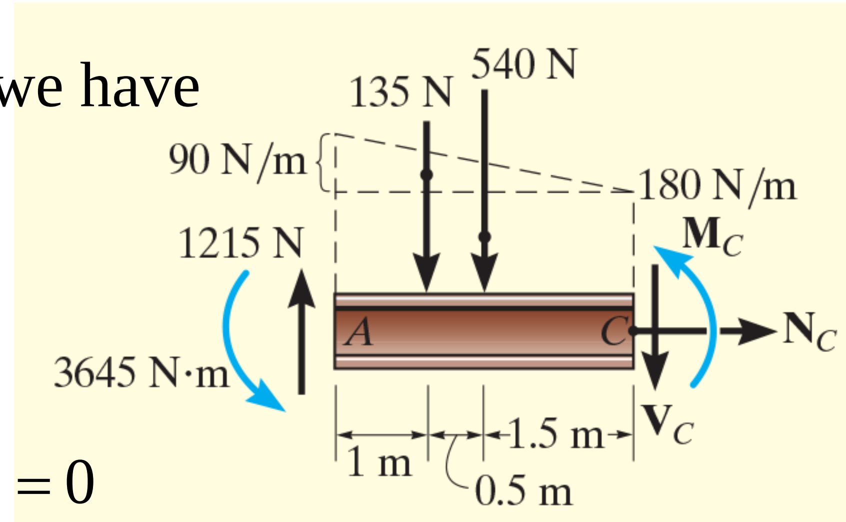
$$F_2 = W_{a1}(s) \times AC = 180 \times 3 = 540 \quad N \text{ acts at ½ of AC } \Rightarrow \frac{1}{2} \times 3 = 1.5 \quad m$$



Example 1

<u>3- Equations of Equilibrium.</u>

Applying the equations of equilibrium we have



$$\rightarrow \sum F_x = 0 \Rightarrow -N_c + N_A = 0$$

$$\Rightarrow N_c = N_A$$

$$\uparrow \sum F_y = 0 \Rightarrow V_A - 540N - 135N - V_C = 0$$

$$\Rightarrow V_A = V_C + 675N = 540N + 675N = 1215N$$

$$\sum M_c = 0 \Rightarrow -M_c - 540N \times 1.5m - 135N \times 1m - 540N \times 3m + M_A = 0$$

$$\Rightarrow M_A = 945 + M_A N.m = 2565 + 1080 = 3645N.m$$