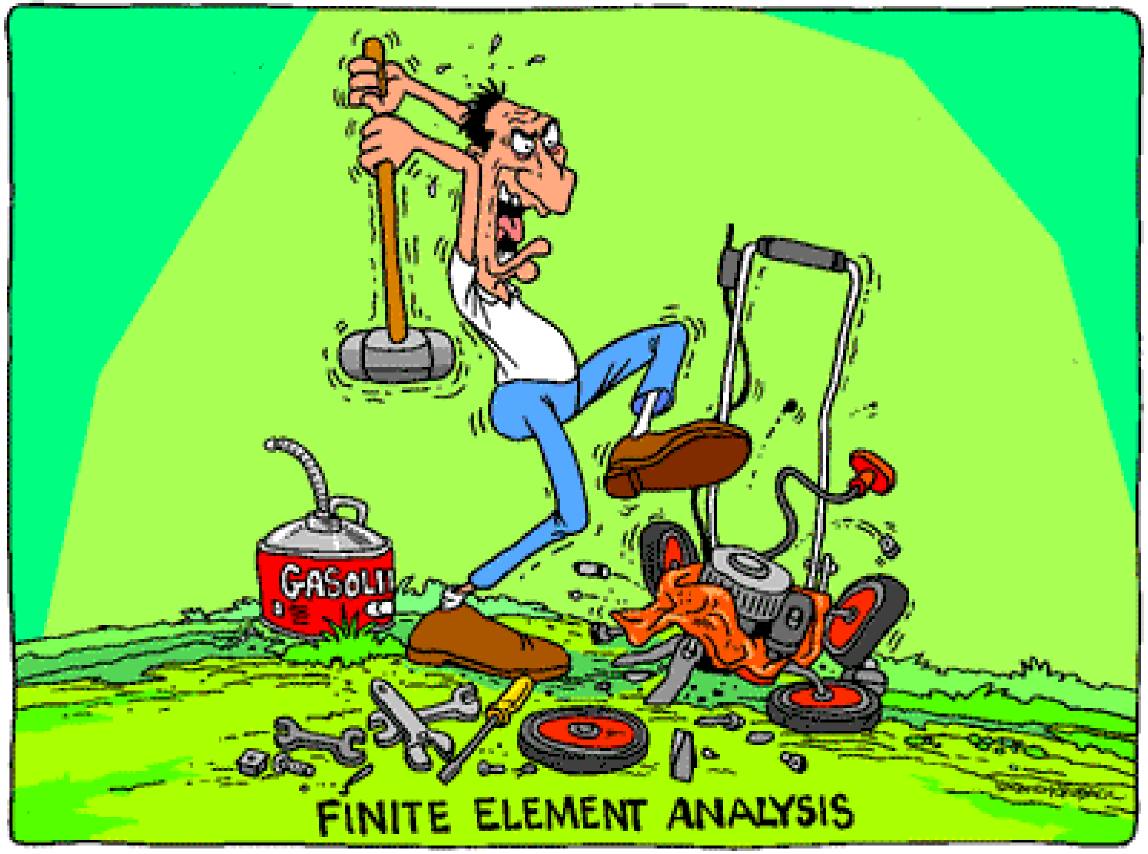
Lecture 13

Finite Element Method (2)

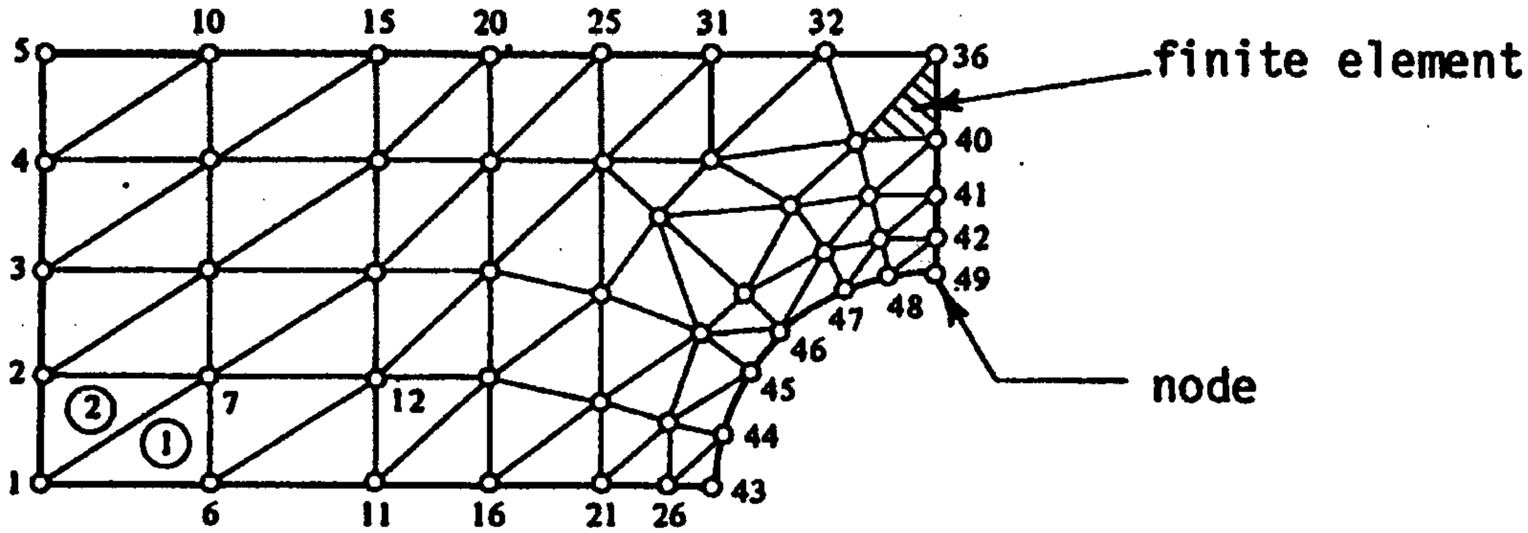
Mohamad Fathi GHANAMEH

Basic Concept of the Finite Element Method



Basic Concept of the Finite Element Method

The fundamental concept involves dividing the body under study into a finite number of pieces (subdomains) called elements. Particular assumptions are then made on the variation of the unknown dependent variable(s) across each element using so-called interpolation or approximation functions.



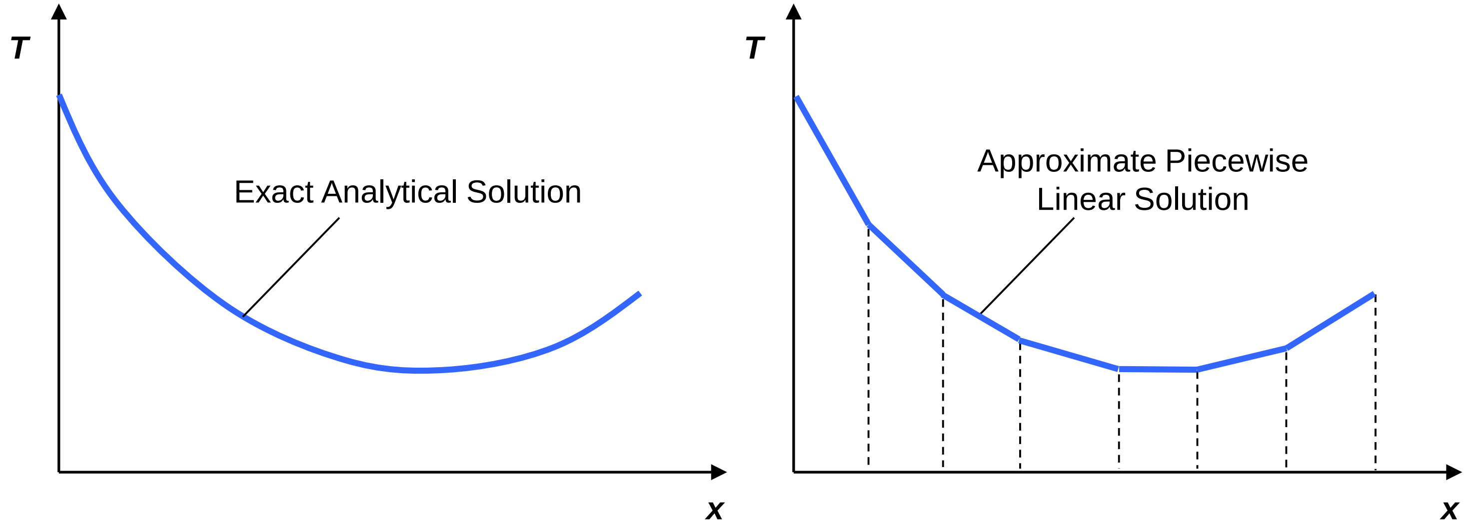
Basic Concept of the Finite Element Method

This approximated variation is quantified in terms of solution values at special element locations called nodes. Through this discretization process, the method sets up an algebraic system of equations for unknown nodal values which approximate the continuous solution. Because element size, shape and approximating scheme can be varied to suit the problem, the method can accurately simulate solutions to problems of complex geometry and loading and thus this technique has become a very useful and practical tool.

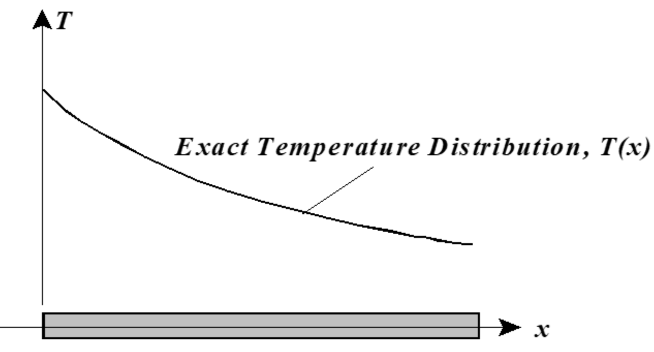
Basic Concept of the Finite Element Method

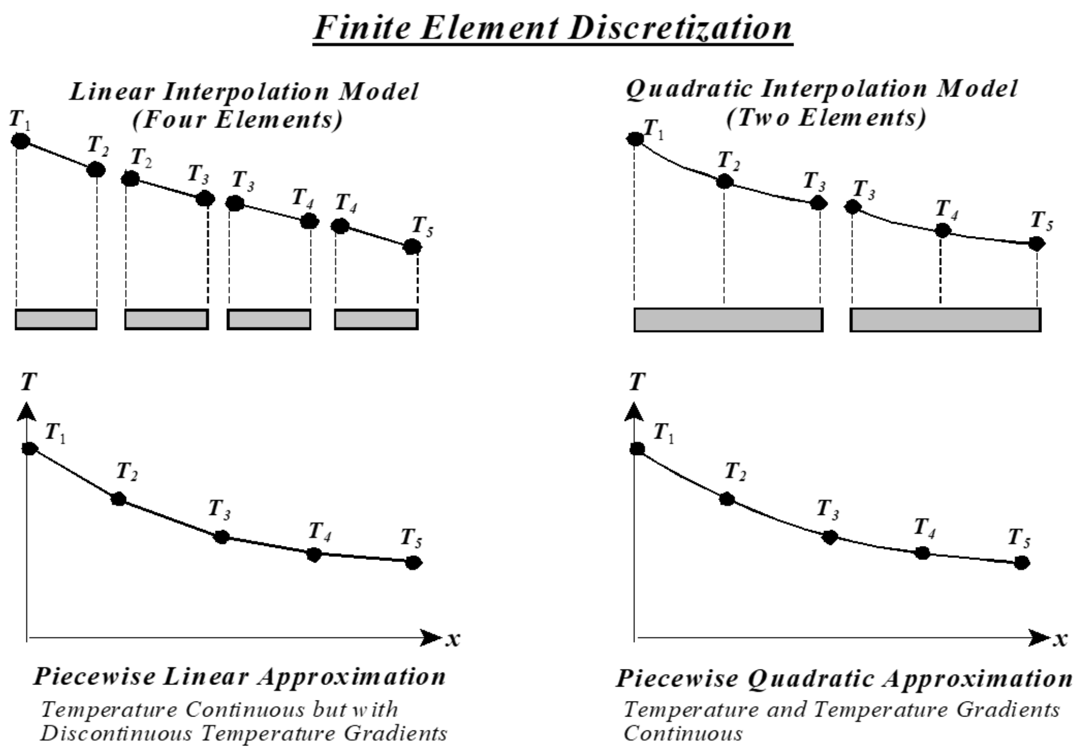
Any continuous solution field such as stress, displacement, temperature, pressure, etc. can be approximated by a discrete model composed of a set of piecewise continuous functions defined over a finite number of subdomains.

One-Dimensional Temperature Distribution



Discretization Concepts

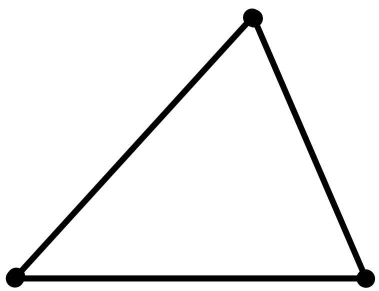


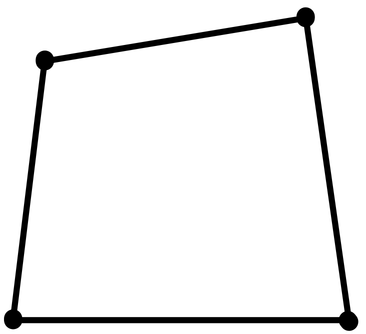


Common Types of Elements

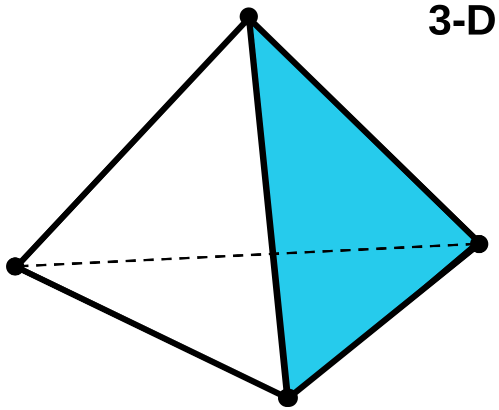
<u>One-Dimensional Elements</u>
Line
Rods, Beams, Trusses, Frames

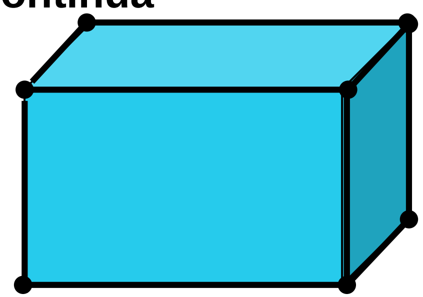
<u>Two-Dimensional Elements</u>
Triangular, Quadrilateral
Plates, Shells, 2-D Continua



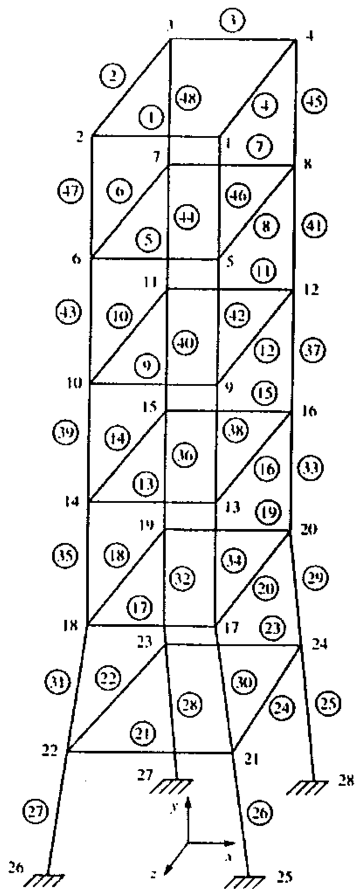


<u>Three-Dimensional Elements</u>
Tetrahedral, Rectangular Prism (Brick)
3-D Continua

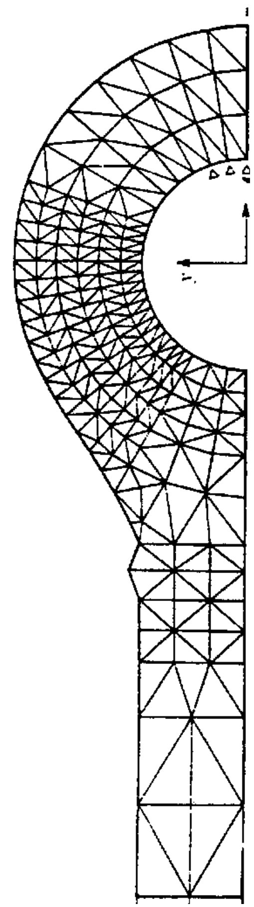




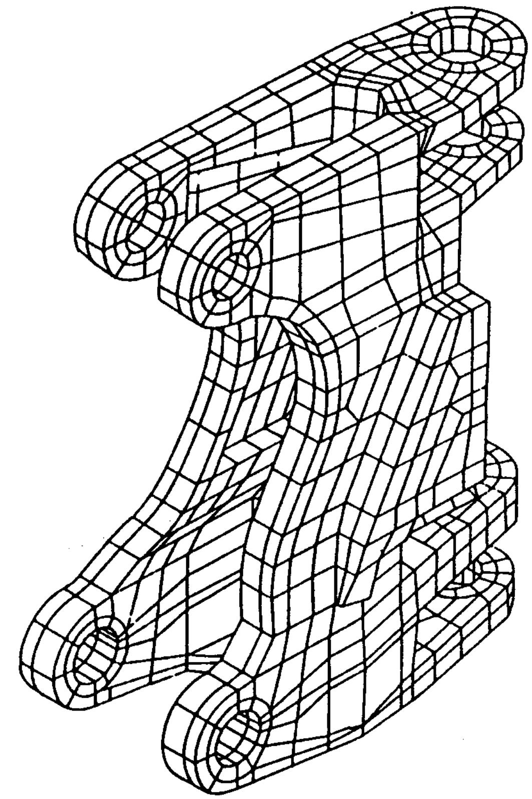
Discretization Examples



One-Dimensional Frame Elements

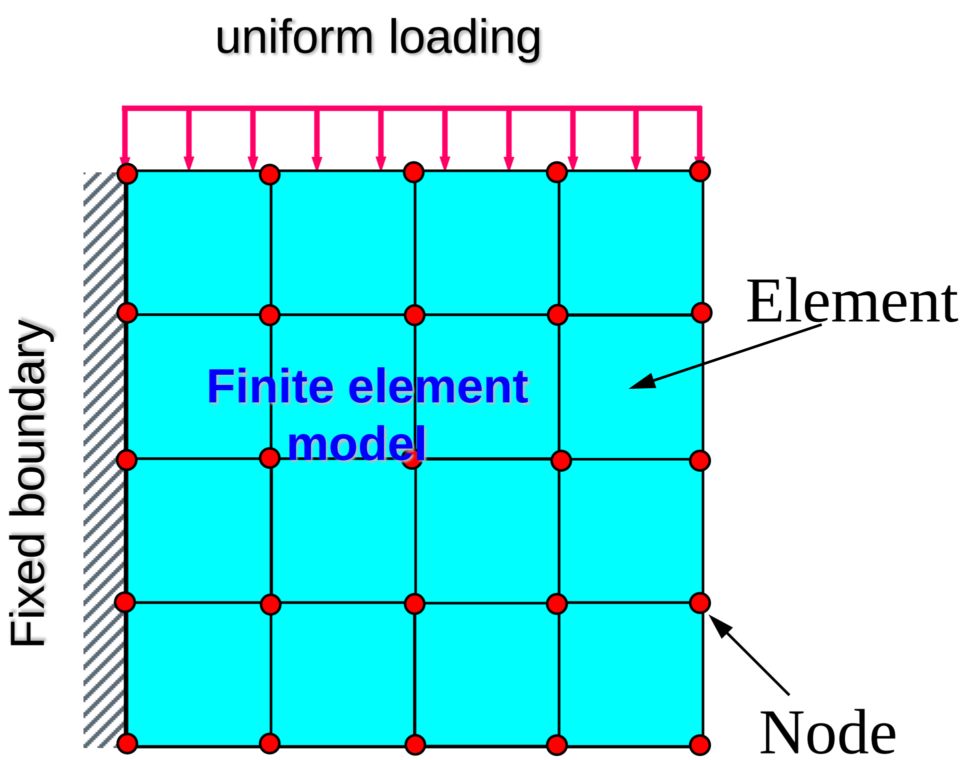


Two-Dimensional Triangular Elements



Three-Dimensional Brick Elements

Finite Element Analysis



- Approximate method
- Geometric model
- Node
- Element
- Mesh
- Discretization

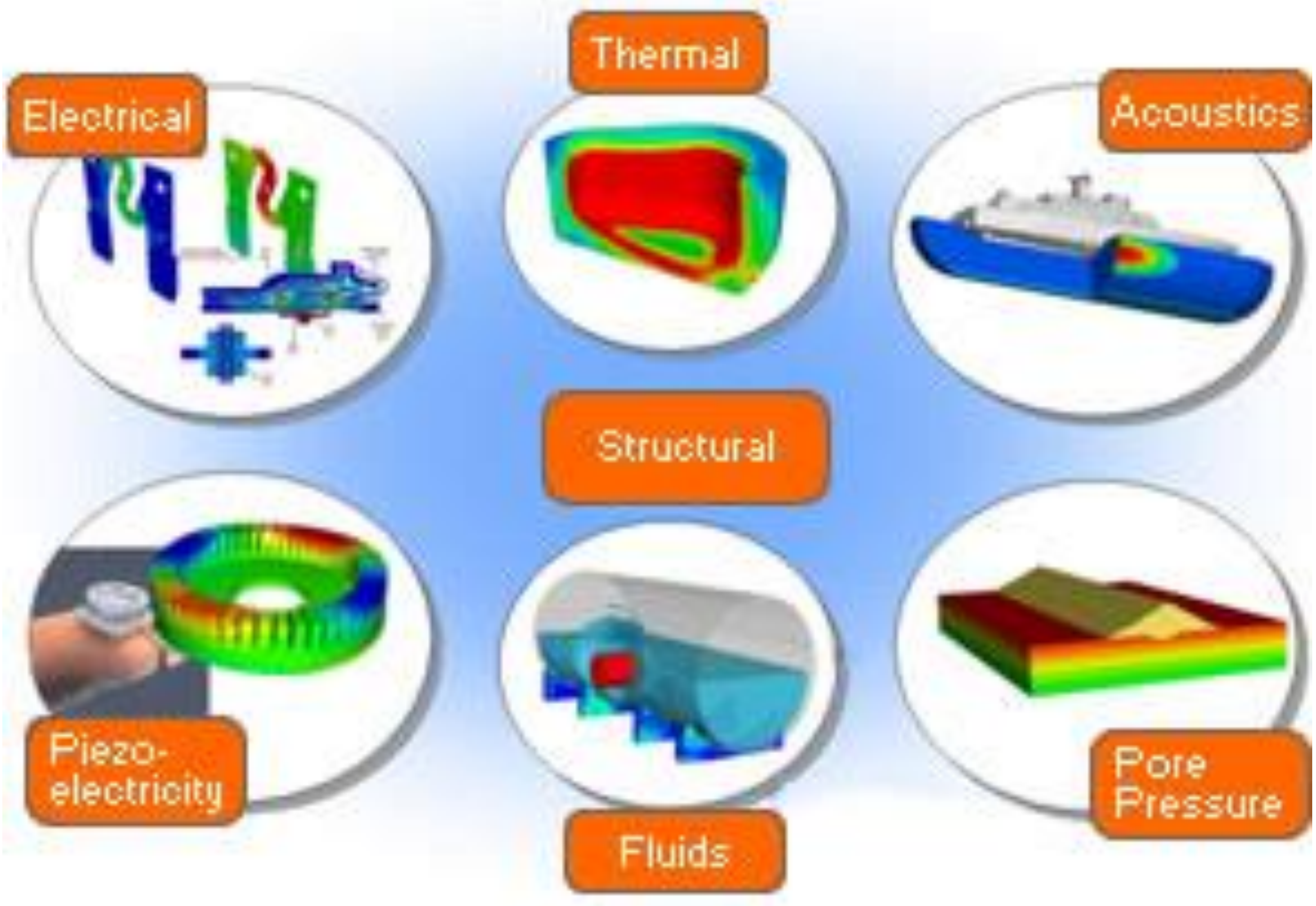
Problem: Obtain the stresses/strains in the plate

Finite Element Analysis

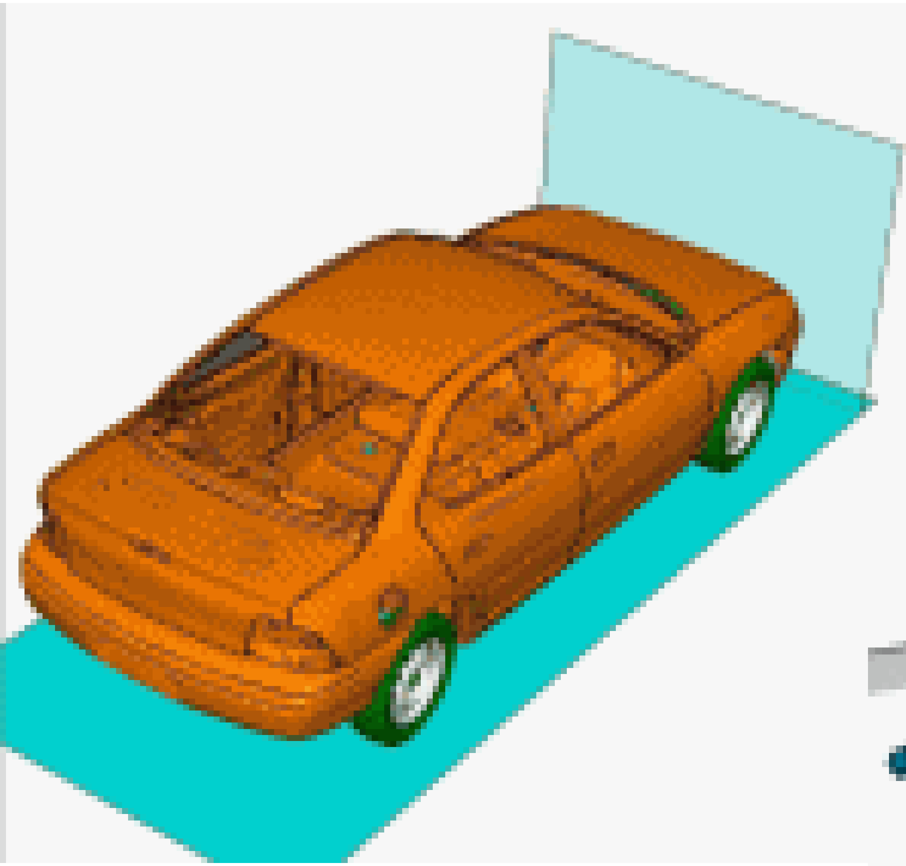
The finite element method is a computational way to solve field problems in engineering and science. The technique has very wide application, and has been used on problems involving:

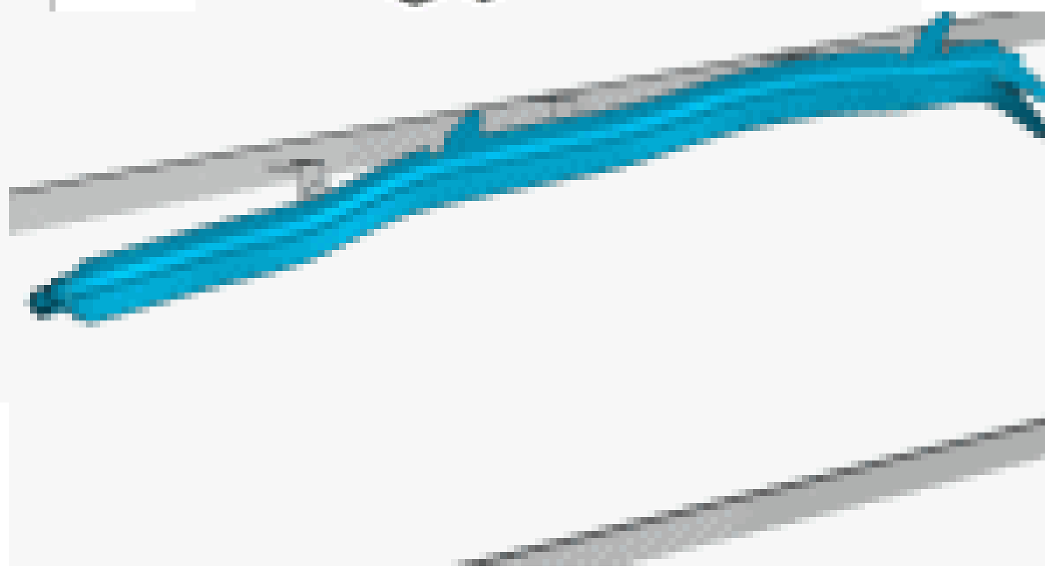
1. stress analysis
2. fluid mechanics
3. heat transfer
4. Diffusion
5. Vibrations
6. Electrical fields
7. magnetic fields

Computational Modelling Using the FEM

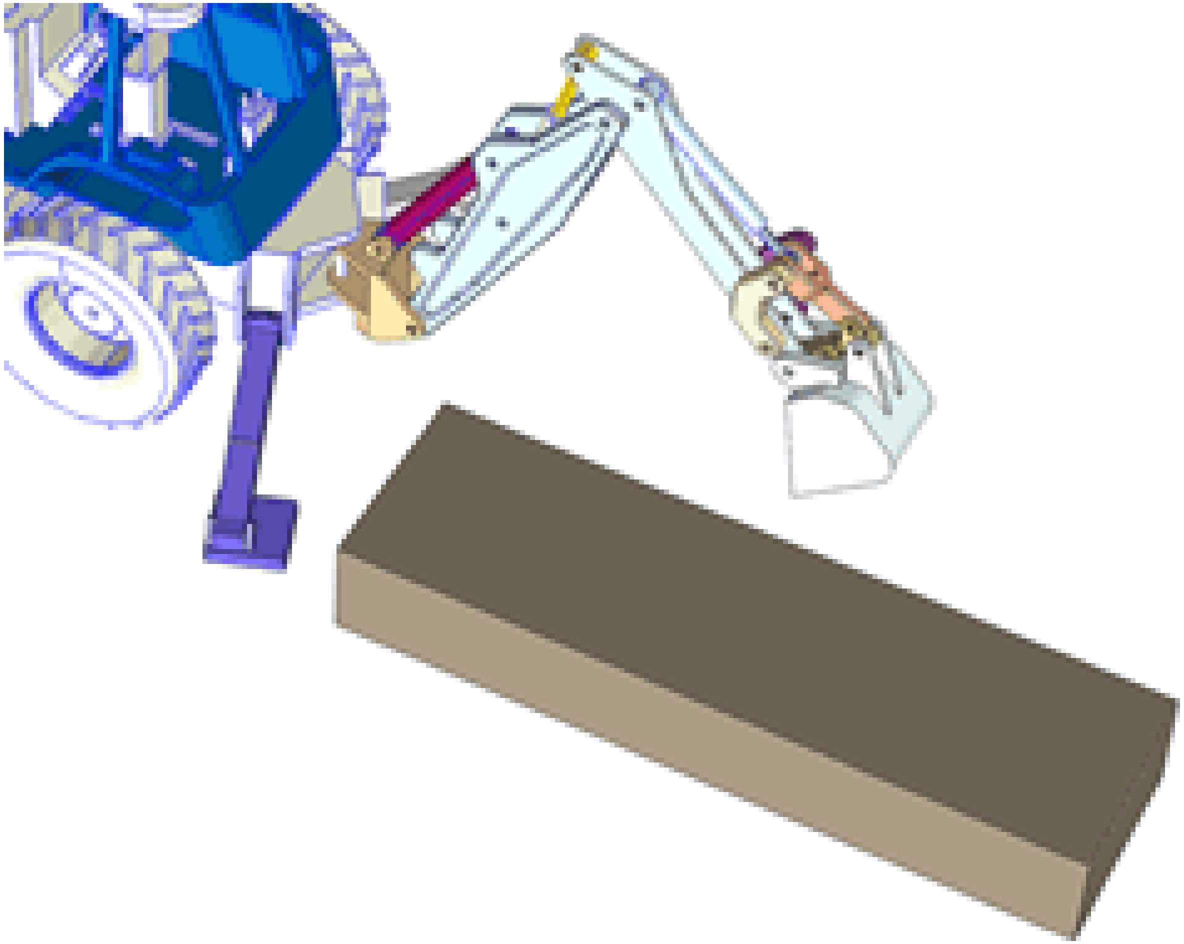


Computational Modelling Using the FEM

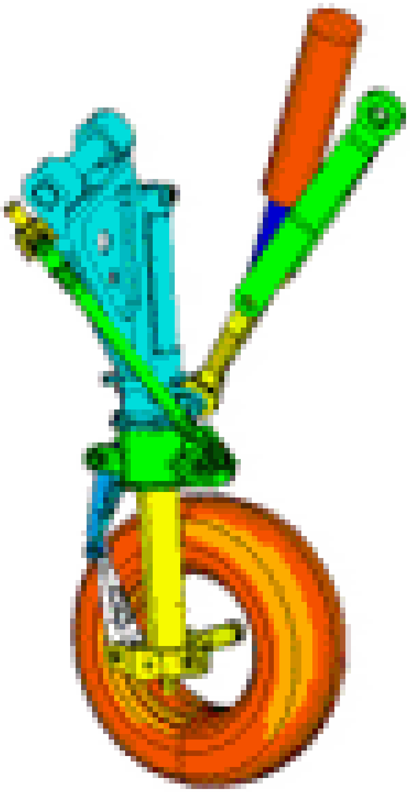




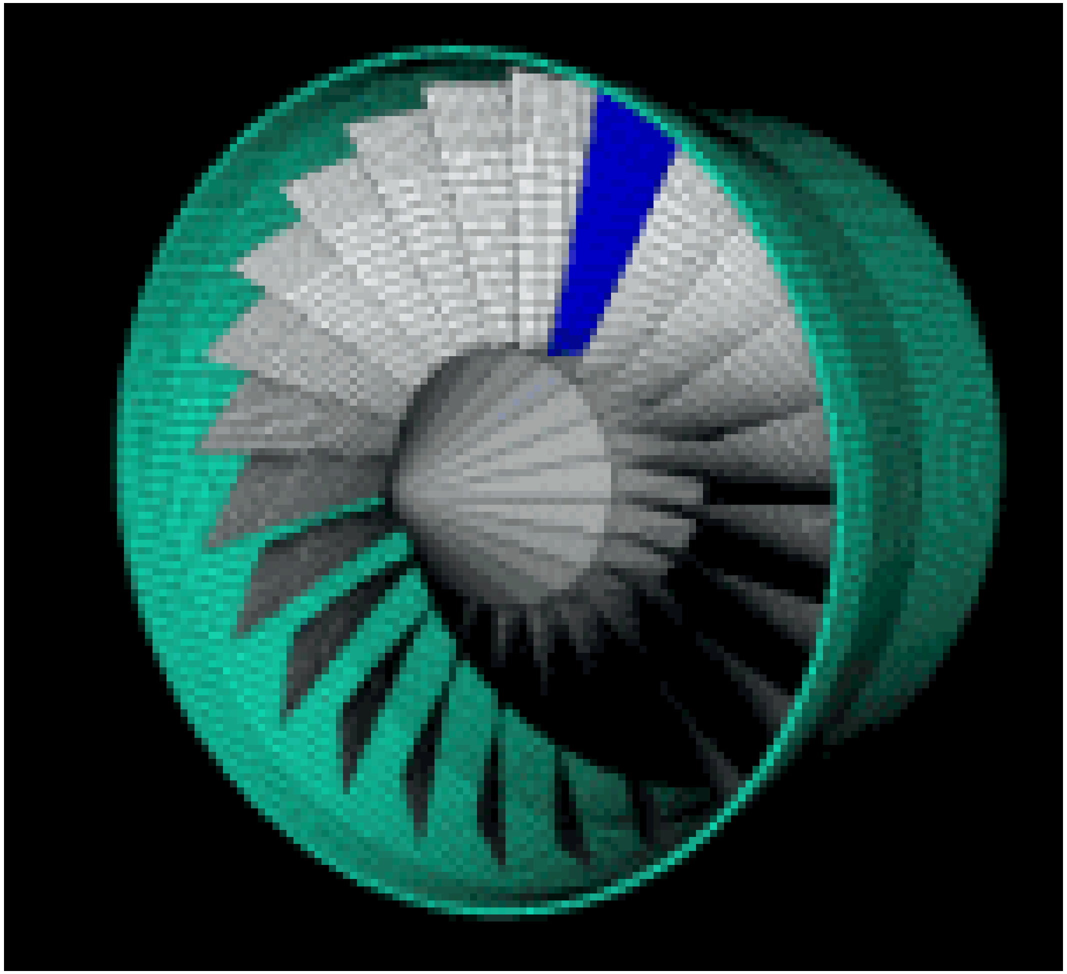
Computational Modelling Using the FEM



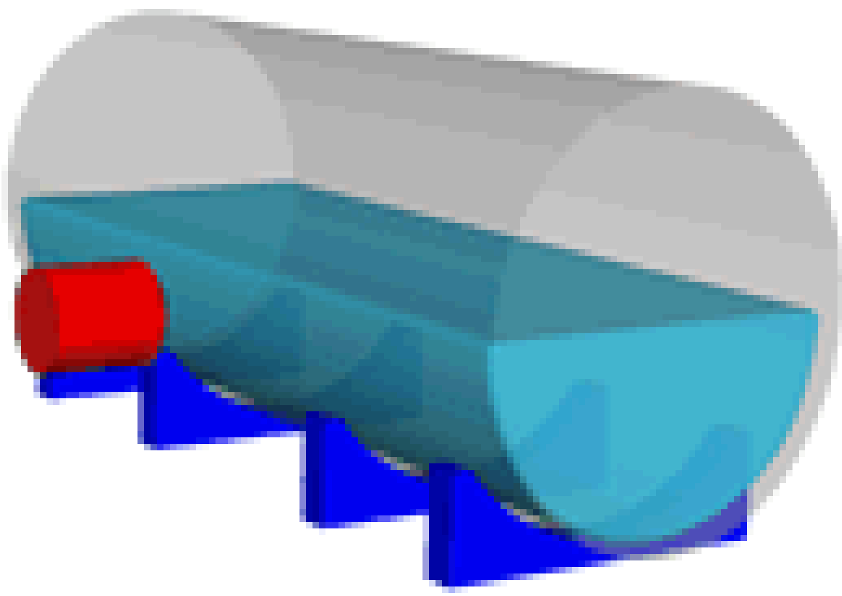
Computational Modelling Using the FEM

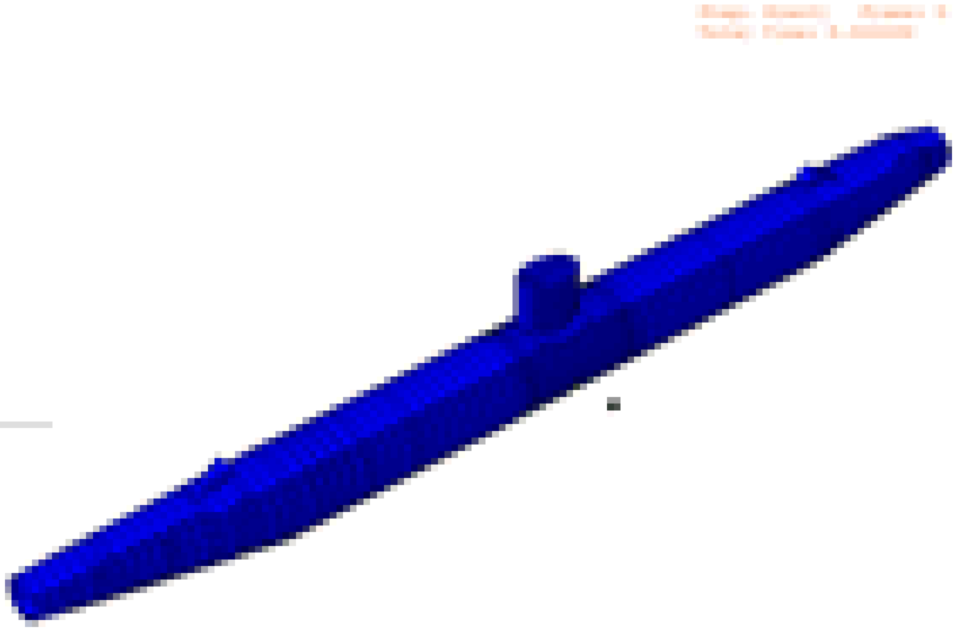


Computational Modelling Using the FEM

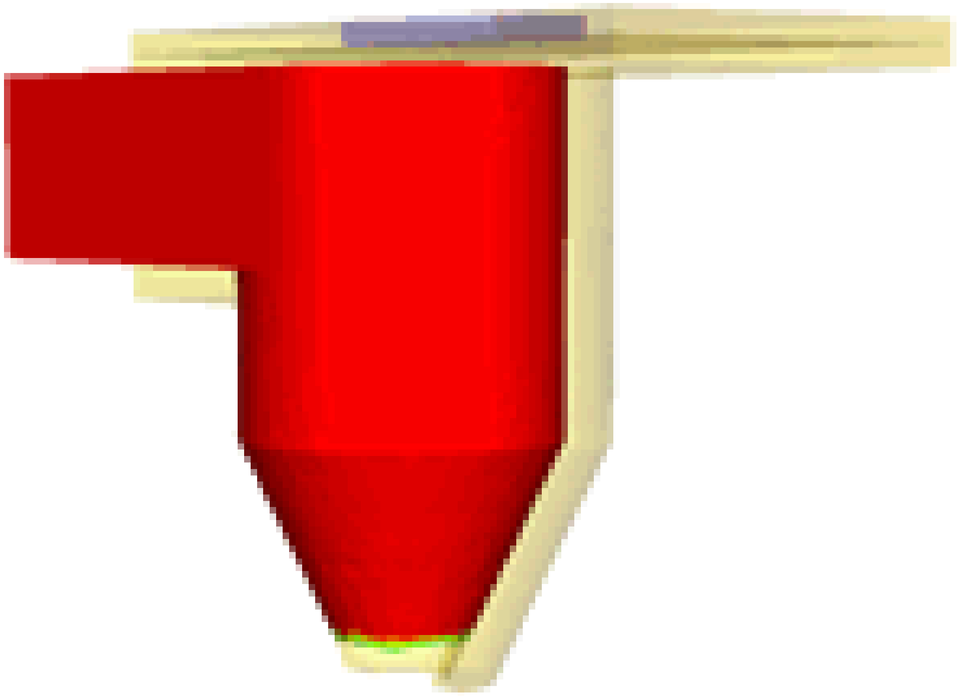


Computational Modelling Using the FEM

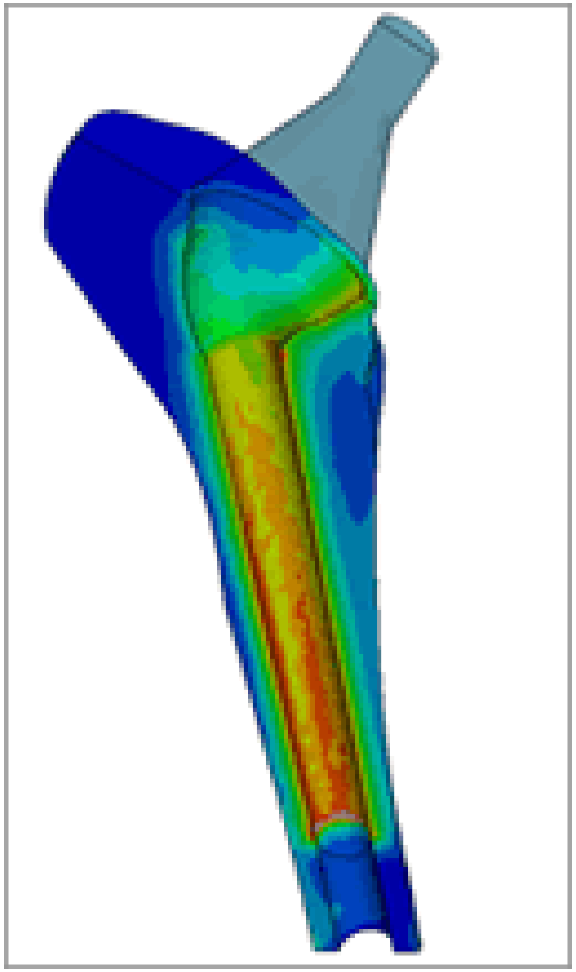


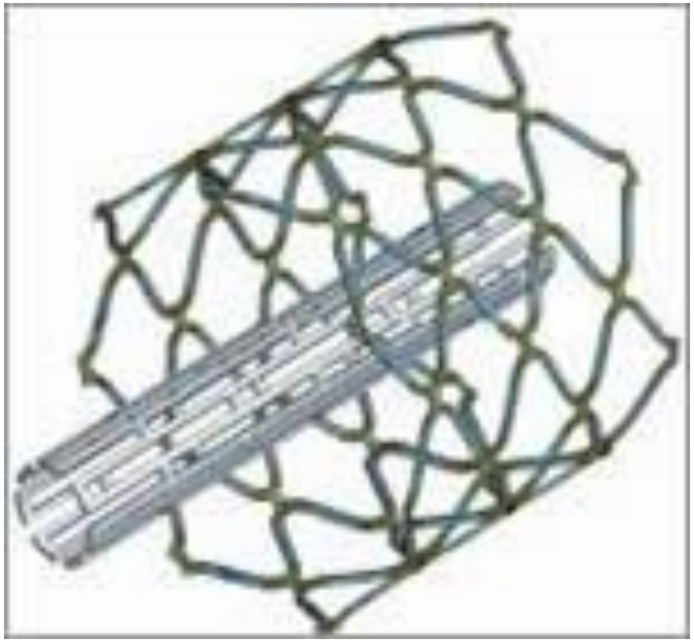


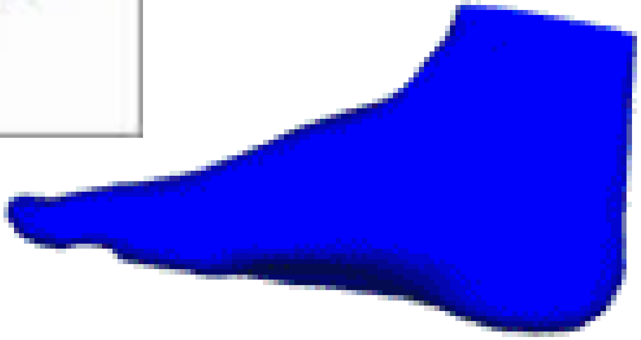
Computational Modelling Using the FEM



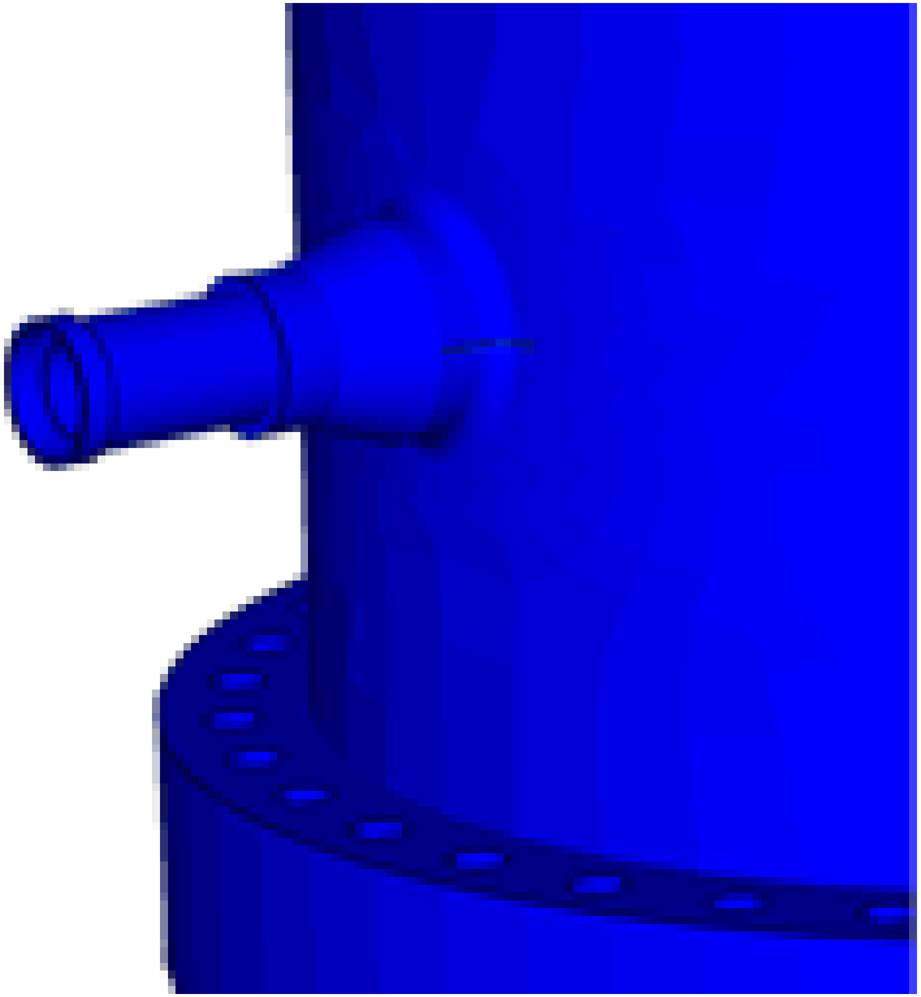
Computational Modelling Using the FEM







Computational Modelling Using the FEM

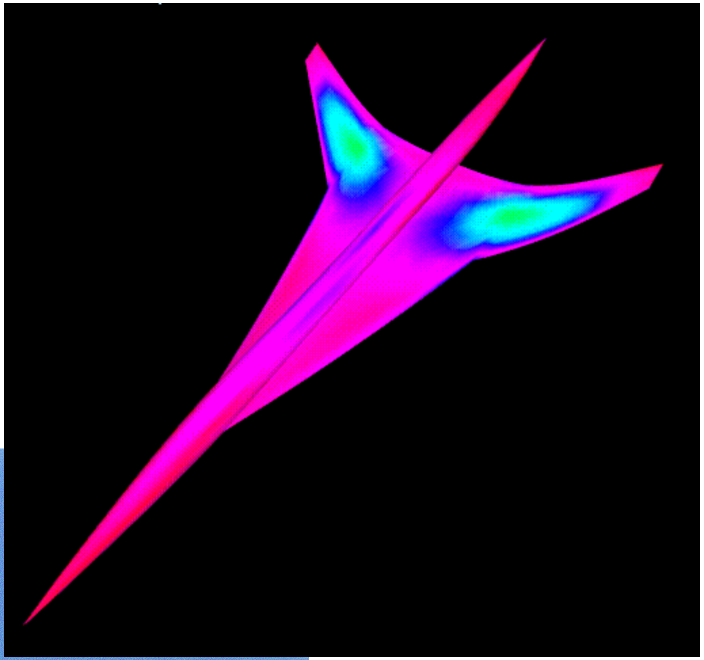


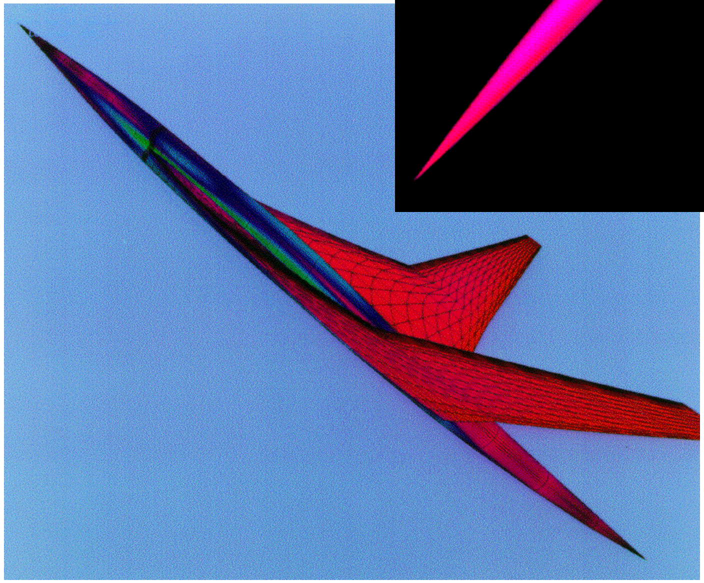
Computational Modelling Using the FEM

Drag Force Analysis of Aircraft

- Question

  What is the drag force distribution on the aircraft?
- Solve
  - Navier-Stokes Partial Differential Equations.
- Recent Developments
  - Multigrid Methods for Unstructured Grids





San Francisco Oakland Bay Bridge

Before the 1989 Loma Prieta earthquake

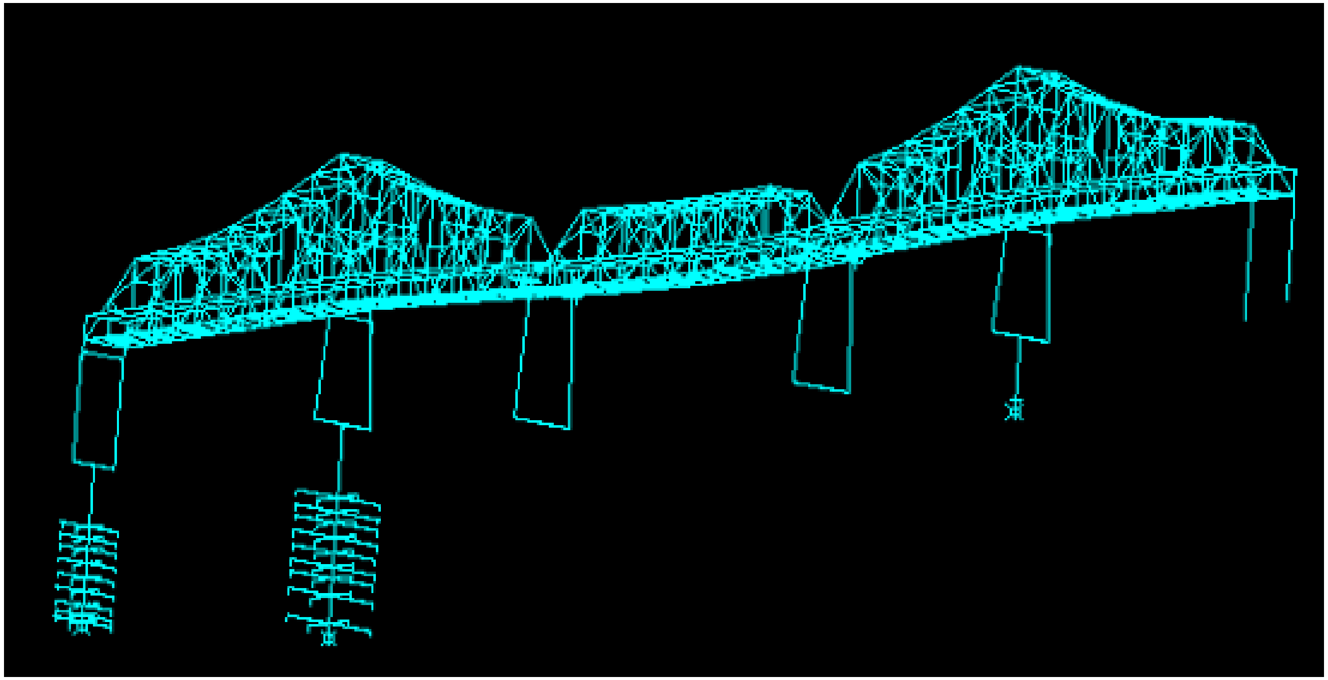
Computational Modelling Using the FEM

San Francisco Oakland Bay Bridge

After the earthquake

Computational Modelling Using the FEM

San Francisco Oakland Bay Bridge



A finite element model to analyze the bridge under seismic loads

Courtesy: ADINA R&D

Computational Modelling Using the FEM

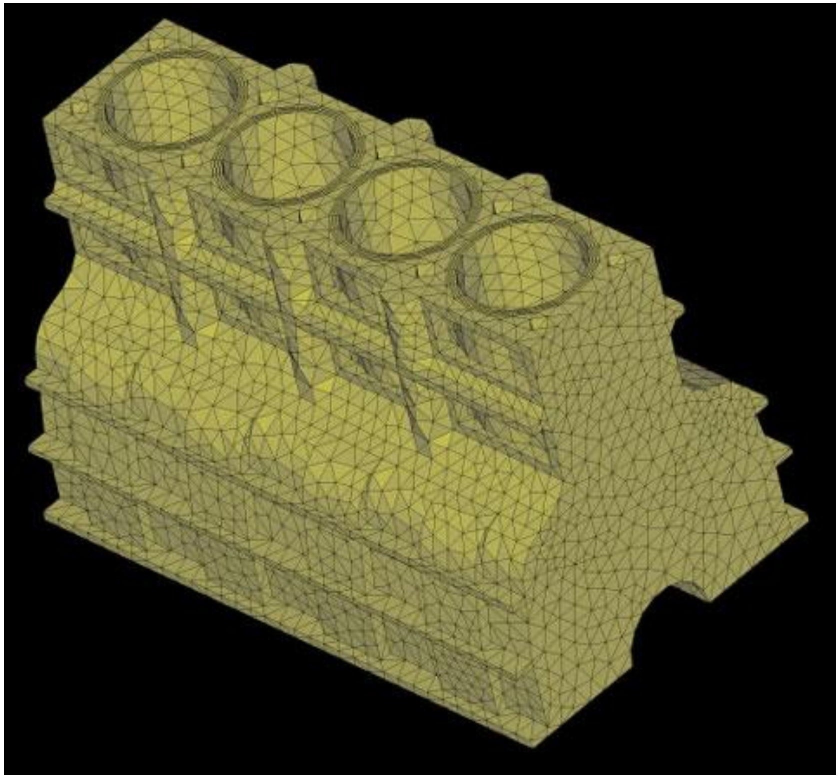
Crush Analysis of Ford Windstar

- Question
  - What is the load-deformation relation?
- Solve
  - Partial Differential Equations of Continuum Mechanics
- Recent Developments
  - Meshless Methods, Iterative methods, Automatic Error Control

Computational Modelling Using the FEM

Engine Thermal Analysis

- Question
  - What is the temperature distribution in the engine block?
- Solve
  - Poisson Partial Differential Equation.
- Recent Developments
  - Fast Integral Equation Solvers, Monte-Carlo Methods

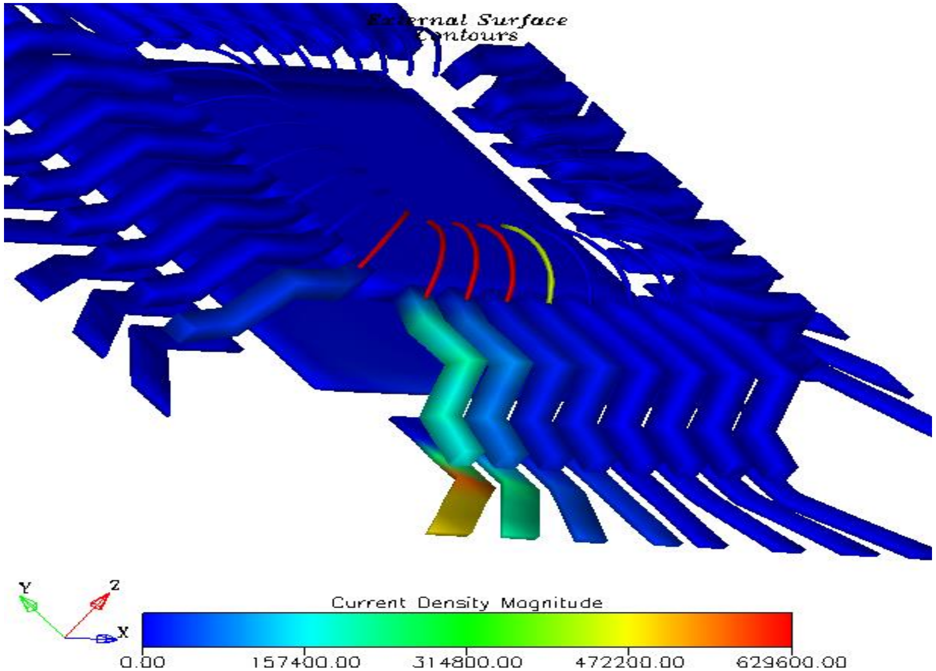


Picture from http://www.adina.com

Computational Modelling Using the FEM

Electromagnetic Analysis of Packages

- Solve
  - Maxwell's Partial Differential Equations
- Recent Developments
  - Fast Solvers for Integral Formulations

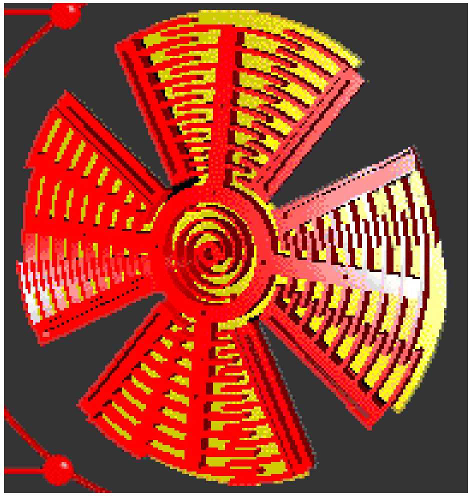


Thanks to Coventor
http://www.coventor.com

Computational Modelling Using the FEM

Micromachine Device   Performance Analysis

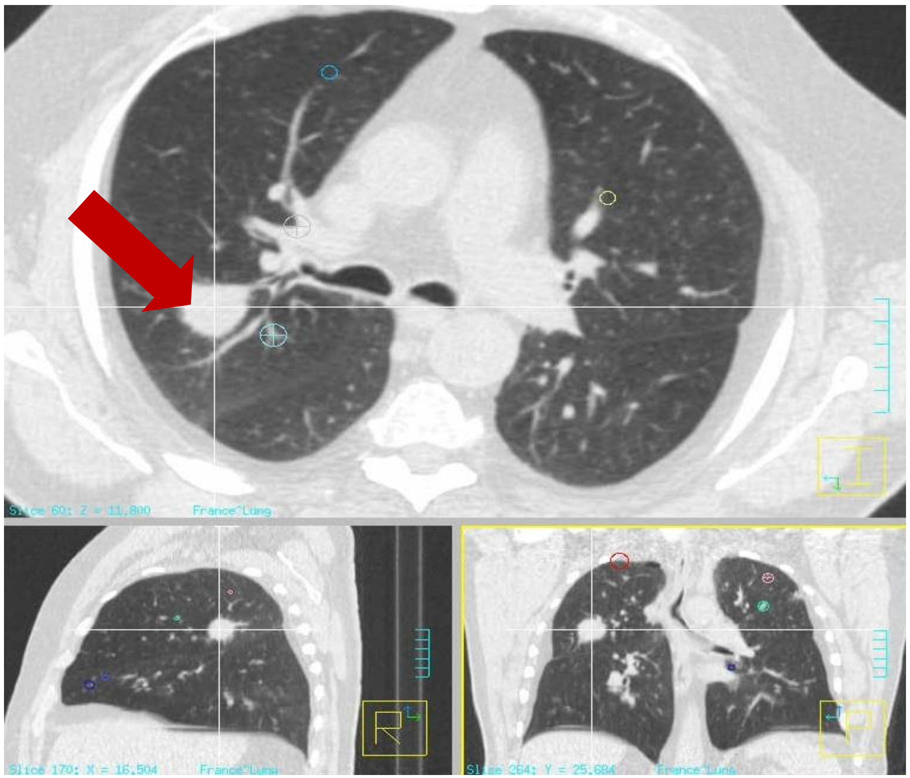
- Equations
  - Elastomechanics, Electrostatics, Stokes Flow.
- Recent Developments
  - Fast Integral Equation Solvers, Matrix-Implicit Multi-level Newton Methods for coupled domain problems.

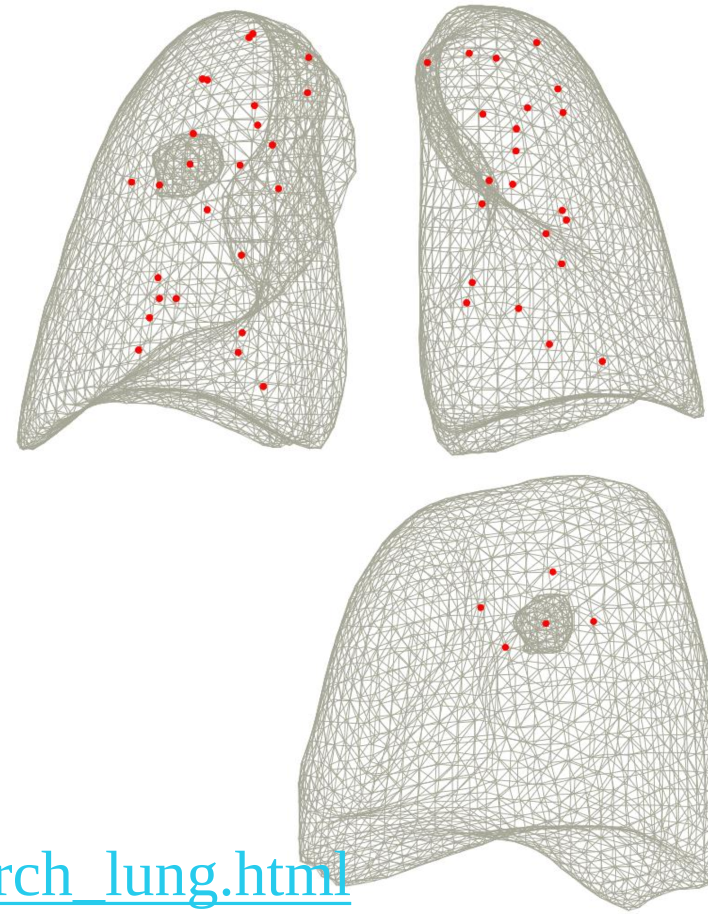


From www.memscap.com

Computational Modelling Using the FEM

Radiation Therapy of Lung Cancer

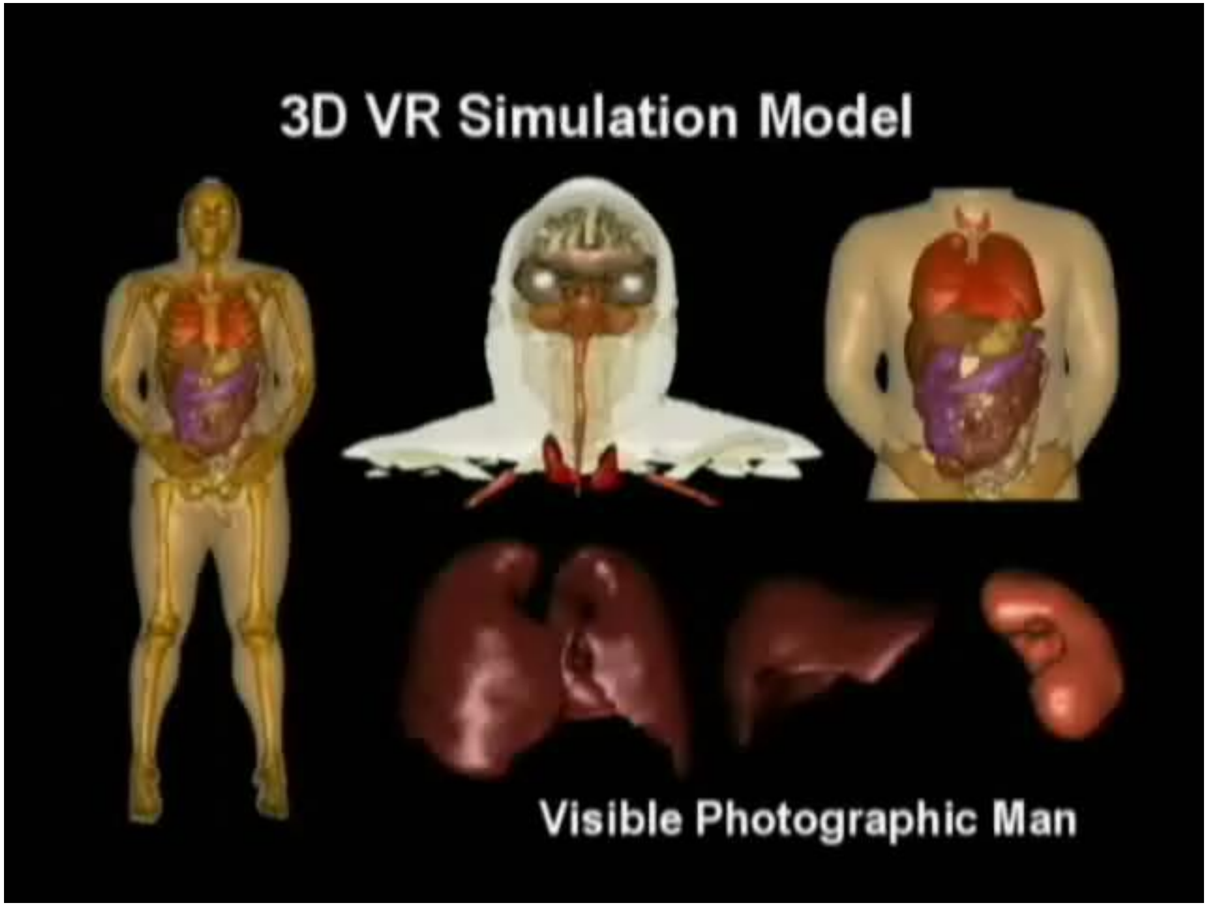




http://www.simulia.com/academics/research_lung.html

Computational Modelling Using the FEM

Virtual Surgery



Advantages of Finite Element Analysis

- ✓ Models Bodies of Complex Shape
- ✓ Can Handle General Loading/Boundary Conditions
- ✓ Models Bodies Composed of Composite and Multiphase Materials
- ✓ Model is Easily Refined for Improved Accuracy by Varying Element Size and Type (Approximation Scheme)
- ✓ Time Dependent and Dynamic Effects Can Be Included
- ✓ Can Handle a Variety Nonlinear Effects Including Material Behavior, Large Deformations, Boundary Conditions, Etc.

Modelling process

Time Independent Problems

- Domain Discretization
- Select Element Type (Shape and Approximation)
- Derive Element Equations (Variational and Energy Methods)
- Assemble Element Equations to Form Global System

$$[K]\{U\} = \{F\}$$

  - [K] = Stiffness or Property Matrix
  - {U} = Nodal Displacement Vector
  - {F} = Nodal Force Vector
- Incorporate Boundary and Initial Conditions
- Solve Assembled System of Equations for Unknown Nodal Displacements and Secondary Unknowns of Stress and Strain Values

Modelling process

-The physical significance of the vectors U and F varies according to the application being modeled

Application Problem	*State (DOF) vector* **u** *represents*	*Conjugate vector* **f** *represents*
Structures and solid mechanics	Displacement	Mechanical force
Heat conduction	Temperature	Heat flux
Acoustic fluid	Displacement potential	Particle velocity
Potential flows	Pressure	Particle velocity
General flows	Velocity	Fluxes
Electrostatics	Electric potential	Charge density
Magnetostatics	Magnetic potential	Magnetic intensity

Modelling process

1. Enter the model
2. Pre-processing: Divide the model into segments/pixels/voxels. You need to know the coordinates of very line and corner.
3. Apply the solver
4. Post-processing: Extract the important parameters (gain, front to back ratio, beam width, bandwidth, input impedance, radiation pattern, near-field strength etc)

Modelling process

The procedure of computational modelling using the FEM broadly consists of four steps:

- Modelling of the geometry.
- Meshing (discretization).
- Specification of material property.
- Specification of boundary, initial and loading conditions.

Common Sources of Error in FEA

- Domain Approximation
- Element Interpolation/Approximation
- Numerical Integration Errors (Including Spatial and Time Integration)
- Computer Errors (Round-Off, Etc.,)

Measures of Accuracy in FEA

Accuracy

Error = |(Exact Solution)-(FEM Solution)|

Convergence

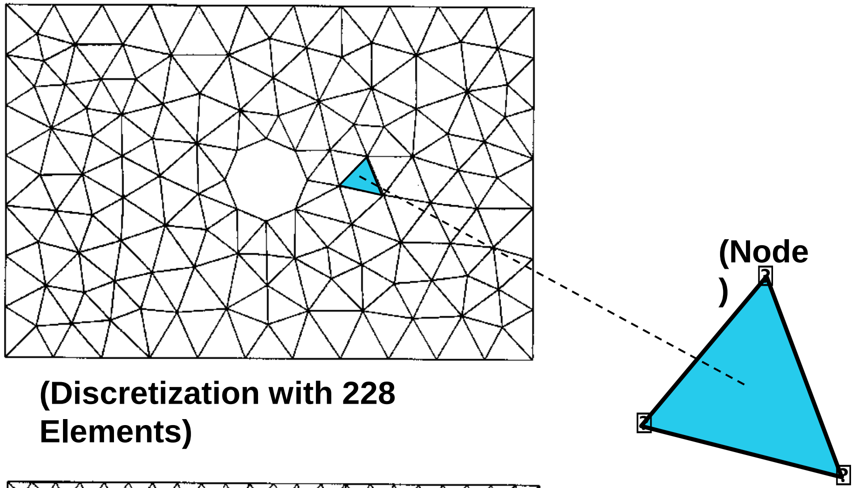
Limit of Error as:

Number of Elements (*h-convergence*)
or
Approximation Order (*p-convergence*)

Increases

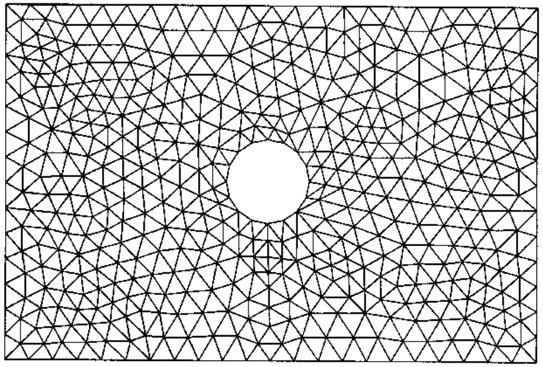
Ideally, Error as Number of Elements or Approximation Order

Two-Dimensional Discretization Refinement



(Discretization with 228 Elements)

(Triangular Element)



(Discretization with 912 Elements)

Computational Modelling Using the FEM

In the following, we will give some examples of finite element applications. The range of applications of

finite elements is too large to list, but to provide an idea of its versatility we list the following:

a. stress and thermal analyses of industrial parts such as electronic chips, electric devices, valves, pipes,

pressure vessels, automotive engines and aircraft;

b. seismic analysis of dams, power plants, cities and high-rise buildings;

c. crash analysis of cars, trains and aircraft;

d. fluid flow analysis of coolant ponds, pollutants and contaminants, and air in ventilation systems;

e. electromagnetic analysis of antennas, transistors and aircraft signatures;

Advantages of the Finite Element Method

They include the ability to

- 1. Model irregularly shaped bodies quite easily
- 2. Handle general load conditions without difficulty
- 3. Model bodies composed of several different materials because the element equations are evaluated individually
- 4. Handle unlimited numbers and kinds of boundary conditions
- 5. Vary the size of the elements to make it possible to use small elements where necessary
- 6. Alter the finite element model relatively easily and cheaply
- 7. Include dynamic effects
- 8. Handle nonlinear behavior existing with large deformations and nonlinear materials

Computer Programs for the FEM

- general-purpose programs are designed to solve many types of problems
- special-purpose programs to solve specific problems

Computer Programs for the FEM

general-purpose programs are designed to solve many types of problems

advantages of general-purpose programs

1. The input is well organized and is developed with user ease in mind. Users do not need special knowledge of computer software or hardware. Preprocessors are readily available to help create the finite element model.
2. The programs are large systems that often can solve many types of problems of large or small size with the same input format.
3. Many of the programs can be expanded by adding new modules for new kinds of problems or new technology. Thus they may be kept current with a minimum of effort.
4. With the increased storage capacity and computational efficiency of PCs, many general-purpose programs can now be run on PCs.
5. Many of the commercially available programs have become very attractive in price and can solve a wide range of problems

Computer Programs for the FEM

general-purpose programs are designed to solve many types of problems

Disadvantages of general-purpose programs

1. The initial cost of developing general-purpose programs is high.
2. General-purpose programs are less efficient than special-purpose programs because the computer must make many checks for each problem, some of which would not be necessary if a special-purpose program were used.
3. Many of the programs are proprietary. Hence the user has little access to the logic of the program. If a revision must be made, it often has to be done by the developers.

Computer Programs for the FEM

special-purpose programs to solve specific problems

advantages of special-purpose programs:

1. The programs are usually relatively short, with low development costs.
2. Small computers are able to run the programs.
3. Additions can be made to the program quickly and at a low cost.
4. The programs are efficient in solving the problems they were designed to solve.

The major disadvantage of special-purpose programs is their inability to solve different classes of problems. Thus one must have as many programs as there are different classes of problems to be solved

Computer Programs for the FEM

There are numerous vendors supporting finite element programs, and the interested user should carefully consult the vendor before purchasing any software. Some existing programs.

Computer Programs for the FEM

- ABAQUS - general-purpose, nonlinear finite element analysis. By HKS.
http://www.simulia.com/
- ADINA - general purpose finite element system for advanced engineering linear and nonlinear analysis of structural, heat transfer, field and fluid flow problems
http://www.adina.com/
- ALG/NASTRAN - complete NASTRAN solution for static stress with linear material models, natural frequency (modal) and steady-state heat transfer analyses, by ALGOR
- ANSYS - FEA software, by ANSYS, Inc.
- FEMdesigner - a basic FEA solver for Microsoft Windows

Computer Programs for the FEM

Ceetron - makers of high-speed 3D visualization and animation software for finite element analysis programs

- COSMOS/M - comprehensive analysis packages includes modeling, meshing, visualization of parts and assemblies from SRAC
- EnSight - post processing for CFD and FEA by CEI
- FEMLAB - finite element modeling and analysis of multidisciplinary problems, by COMSOL
- ALGOR - analysis and simulation tools including static stress, Mechanical Event Simulation (MES) with linear and nonlinear material models, linear dynamics, steady-state and transient heat transfer, steady and unsteady fluid flow, electrostatics, full multiphysics and piping

Computer Programs for the FEM

•LapFEA - imports, exports and solves existing NASTRAN and pal 2 models, and performs 3D stress and vibration analysis of structures, components and mechanical systems. Runs on both Windows and Macintosh OS. From LapCAD Engineering.

•LS-DYNA - nonlinear dynamic finite element software, from Livermore Software Technology Corporation

•MSC.visualNastran - a group of FEA products that includes MSC.Marc, a nonlinear simulation technology for manufacturing applications; MSC.Nastran, an analysis tool for optimized designs; and MSC.Patran, a finite element modeler. From MSC.Software

Computer Programs for the FEM

- NEiNastran - based on NASA's NASTRAN FEA software and designed for personal computers, it offers many analysis types, including linear and nonlinear statics, dynamics and heat transfer. From Noran Engineering, Inc.
- NX Nastran - UGS version of MSC.Nastran software
- NISA/Display - general and special purpose finite element programs for PCs, workstations & supercomputers by EMRC
- Pro/MECHANICA - design simulation tool for motion, structural, thermal and vibration analysis. From PTC.

Computer Programs for the FEM

- SAMCEF - general solver with modules for linear, non-linear, and thermal analyses, by Samtech
- Strand7 - Windows based FEA with automatic meshing of CAD models, linear and nonlinear static, dynamic, heat transfer solvers, postprocessing and API
- FlexPDE - 2D and 3D field analysis such as electromagnetics, heat flow and chemical reactions