Lecture 12

Finite Element Method (1)

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Exploratory Methods In Engineering

- Analytical method
- Experimental method
- Numerical method

Mathematical Models

- A formulation or equation that expresses the essential features of a physical system or process in mathematical terms.
- Models can be represented by a functional relationship between:
  - dependent variables,
  - independent variables,
  - parameters, and
  - forcing functions.

$$\begin{matrix}\text{Dependent}\\ \text{variable}\end{matrix} = f\left(\begin{matrix}\text{independent}\\ \text{variables}\end{matrix}, \text{parameters}, \begin{matrix}\text{forcing}\\ \text{functions}\end{matrix}\right)$$

- Once a mathematical model is constructed one could use
  - Analytical methods
  - Numerical methods

Analytical methods

- Produce exact solutions
- Not always feasible
- May require mathematical sophistication
- The time to solve the problem is a function of its complexity.
- Not Expensive
- Safe

Numerical methods

- Produce an approximate solution
- Mostly feasible
- The time to solve a numerical problem is a function of the desired accuracy of the approximation.
- Not Expensive
- Safe

Experimental methods

- Produce exact solutions
- Not always feasible
- Time consuming
- Expensive
- Measurement difficulties (cables, calibration, mechanical precision, interference, mutual coupling, field confinement, etc)
- Not safe some times

Why numerical modelling?

- To reduce the number of prototypes constructed.
- To use as the forward solver in an optimization routine to gain the best possible design.
- To save the time consuming
- To save money
- Even to save life

But for Verification ….

- You MUST make a physical model!
- You MUST undertake some simple theoretical modelling for use in your numerical modelling analysis!

- If you don’t, you have no verification of your model!
- This is true EVEN if you use “proven” commercial software.

Need for Computational Methods

Solutions using either strength of materials or theory of elasticity are normally accomplished for regions and loadings with relatively simple geometry

Many applications involve cases with complex shape, boundary conditions and material behavior

Therefore a gap exists between what is needed in applications and what can be solved by analytical closed-form methods

This has lead to the development of several numerical/computational schemes including: Finite Difference, Finite Element and Boundary Element Methods

Computational Modelling

Finite Element Method (FEM)

often used for solids and structures

energy principles, such as Hamilton's principle, the minimum potential energy principle

weighted residual method, for heat transfer problems

Computational Modelling Using the FEM

Finite Difference Method (FDM)

Often used for fluid flow simulation it can be also used for solids and structures

Taylor series

Computational Modelling Using the FEM

Finite Volume Method (FVM)

Often used for fluid flow simulation

control of conservation laws on each finite volume (elements) in the domain