Structural Analysis

Lecture 10

Stiffness Method (2)

For truss analysis

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Application of the stiffness method for truss analysis

The global force components Q acting on the truss can then be related to its global displacements D using

$$Q = KD$$

This equation is referred to as the structure stiffness equation

$$\left[\begin{array}{c}\mathbf{Q}_k \\ \hdashline \mathbf{Q}_u\end{array}\right] = \left[\begin{array}{c:c}\mathbf{K}_{11} & \mathbf{K}_{12} \\ \hdashline \mathbf{K}_{21} & \mathbf{K}_{22}\end{array}\right]\left[\begin{array}{c}\mathbf{D}_u \\ \hdashline \mathbf{D}_k\end{array}\right]$$

Application of the stiffness method for truss analysis

Expanding yields

$$Q_k = K_{11}D_u + K_{12}D_k$$
$$Q_u = K_{21}D_u + K_{22}D_k$$

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Often $D_k = 0$ since the supports are not displaced
Thus becomes

$$Q_k = K_{11} D_u$$

Application of the stiffness method for truss analysis

Since the elements in the partitioned matrix K_{11} represent the total resistance at a truss joint to a unit Displacement in either the x or y direction, then the above equation symbolizes the collection of all the force equation applied to the joints where the external loads are zero or have a known value Q_k Solving for D_u, we have:

$$D_u = [K_{11}]^{-1} Q_k$$

Application of the stiffness method for truss analysis

Expanding yields

$$Q_k = K_{11}D_u + K_{12}D_k$$
$$Q_u = K_{21}D_u + K_{22}D_k$$

Often $D_k = 0$ since the supports are not displaced
Thus becomes

$$Q_u = K_{21}D_u$$

Application of the stiffness method for truss analysis

The member forces can be determined

$$\begin{bmatrix} q_N \\ q_F \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} d_N \\ d_F \end{bmatrix} \qquad \begin{bmatrix} d_N \\ d_F \end{bmatrix} = \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix}\begin{bmatrix} D_{N_x} \\ D_{N_y} \\ D_{F_x} \\ D_{F_y} \end{bmatrix}$$

$$\begin{bmatrix} q_N \\ q_F \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_x & \lambda_y \end{bmatrix}\begin{bmatrix} D_{N_x} \\ D_{N_y} \\ D_{F_x} \\ D_{F_y} \end{bmatrix}$$

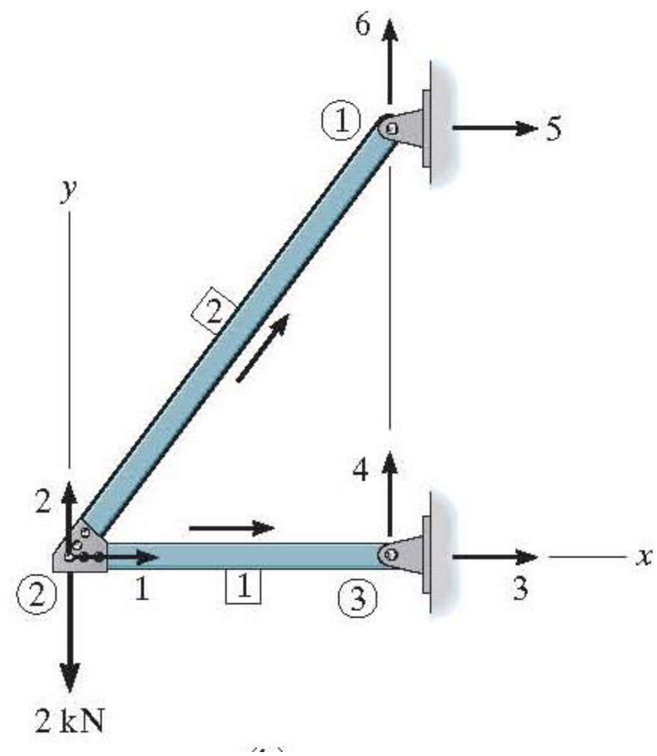
Application of the stiffness method for truss analysis

Since with qN = -qF :

$$q_F = \frac{AE}{L}\begin{bmatrix} -\lambda_x & -\lambda_y & \lambda_x & \lambda_y \end{bmatrix}\begin{bmatrix} D_{N_x} \\ D_{N_y} \\ D_{F_x} \\ D_{F_y} \end{bmatrix}$$

Example 1

Determine the force in each member of the 2-member truss as shown. AE is constant.



Solution

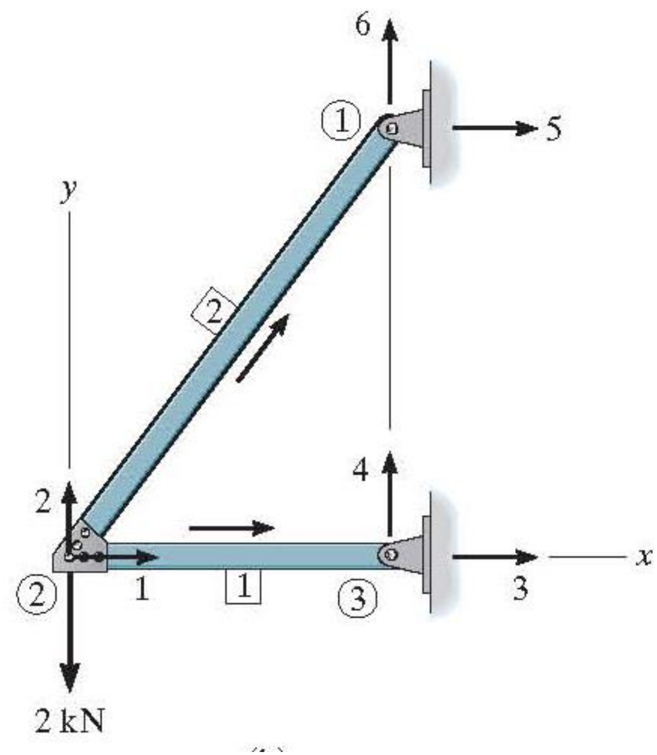
The origin of x,y and the numbering of the joints & members are shown.

By inspection, it is seen that the known external displacement are D3=D4=D5=D6=0

Also, the known external loads are Q1=0, Q2=-2kN.

Hence,

$$D_k = \begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array} \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \end{array} \qquad Q_k = \begin{array}{c} \begin{bmatrix} 0 \\ -2 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$



Solution

Using the same notation as used here, this matrix has been developed before.

Writing equation Q = KD for the truss we have

$$\begin{bmatrix} 0 \\ -2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = AE \left[\begin{array}{cc:cccc} 0.405 & 0.096 & -0.333 & 0 & -0.072 & -0.096 \\ 0.096 & 0.128 & 0 & 0 & -0.096 & -0.128 \\ \hdashline -0.333 & 0 & 0.333 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -0.072 & -0.096 & 0 & 0 & 0.072 & 0.096 \\ -0.096 & -0.128 & 0 & 0 & 0.096 & 0.128 \end{array}\right] \begin{bmatrix} D_1 \\ D_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution

We can now identify K11 and thereby determine Du
By matrix multiplication,

$$\begin{bmatrix} 0 \\ -2 \end{bmatrix} = AE\begin{bmatrix} 0.405 & 0.096 \\ 0.096 & 0.128 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

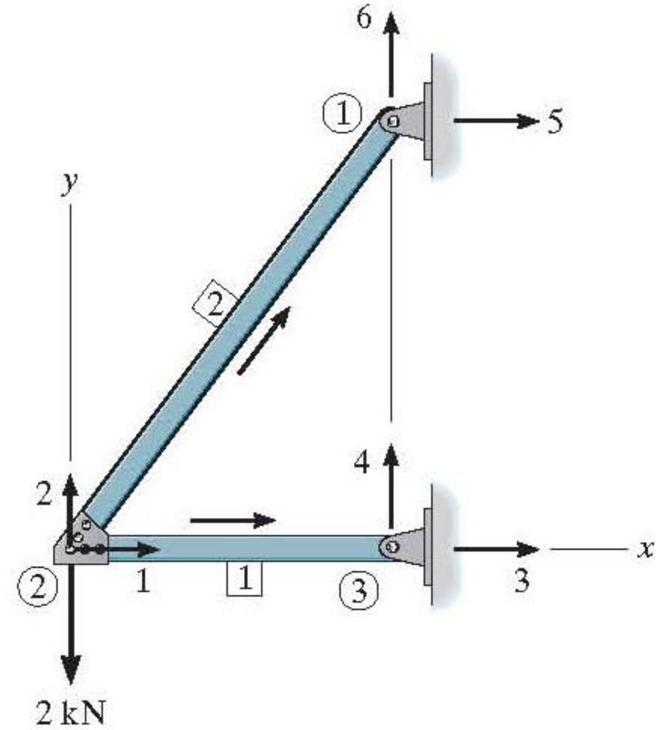
$$D_1 = \frac{4.505}{AE}; \quad D_2 = \frac{-19.003}{AE}$$

Solution

By inspection one would expect a rightward and downward Displacement to occur at joint 2 as indicated by the +ve & -ve signs of the answers.

Using these results,

$$\begin{bmatrix} Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = AE \begin{bmatrix} -0.333 & 0 \\ 0 & 0 \\ -0.072 & -0.096 \\ -0.096 & -0.128 \end{bmatrix} \frac{1}{AE} \begin{bmatrix} 4.505 \\ -19.003 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



Solution

Expanding & solving for the reactions

$$Q_3 = -1.5kN$$
$$Q_4 = 0kN$$
$$Q_5 = 1.5kN$$
$$Q_6 = 2.0kN$$

The force in each member can be found.

Using the data for λx and λy in example 1, we have:

For member 1,

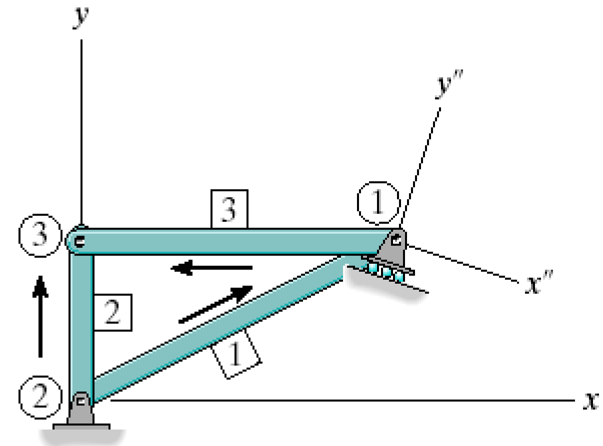
$$\lambda_x = 1, \quad \lambda_y = 0, \quad L = 3m \Rightarrow q_1 = -1.5kN$$

For member 2,

$$\lambda_x = 0.6, \quad \lambda_y = 0.8, \quad L = 5m \Rightarrow q_2 = 2.5kN$$

Nodal Coordinates

- A truss can be supported by a roller placed on a incline
- When this occurs, the constraint of zero deflection at the support (node) cannot be directly defined using a single horizontal & vertical global coordinate system
- Consider the truss
- The condition of zero displacement at node 1 is defined only along the y” axis



Nodal Coordinates

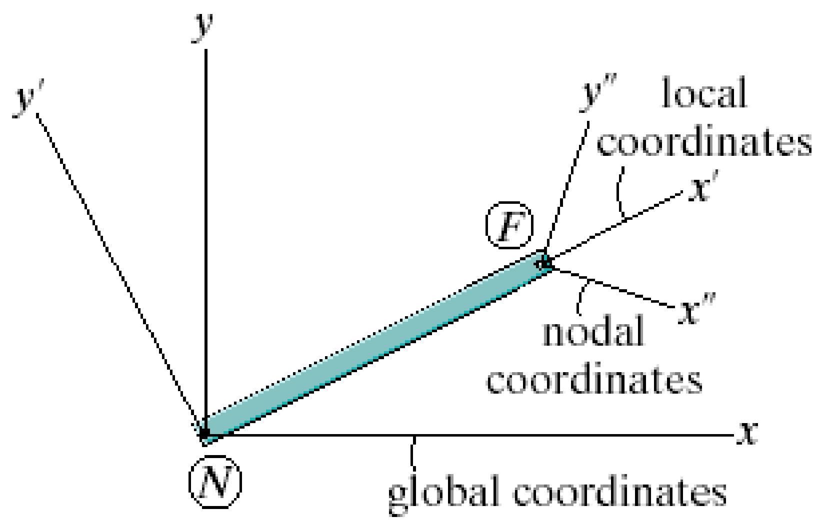
- Because the roller can displace along the x” axis this node will have displacement components along both global coordinates axes x & y
- To solve this problem, so that it can easily be incorporated into a computer analysis, we will use a set of nodal coordinates x”, y” located at the inclined support
- These axes are oriented such that the reactions & support displacement are along each of the coordinate axes

Nodal Coordinates

- To determine the global stiffness equation for the truss, it becomes necessary to develop force & displacement transformation matrices for each of the connecting members at this support so that the results can be summed within the same global x, y coordinate system

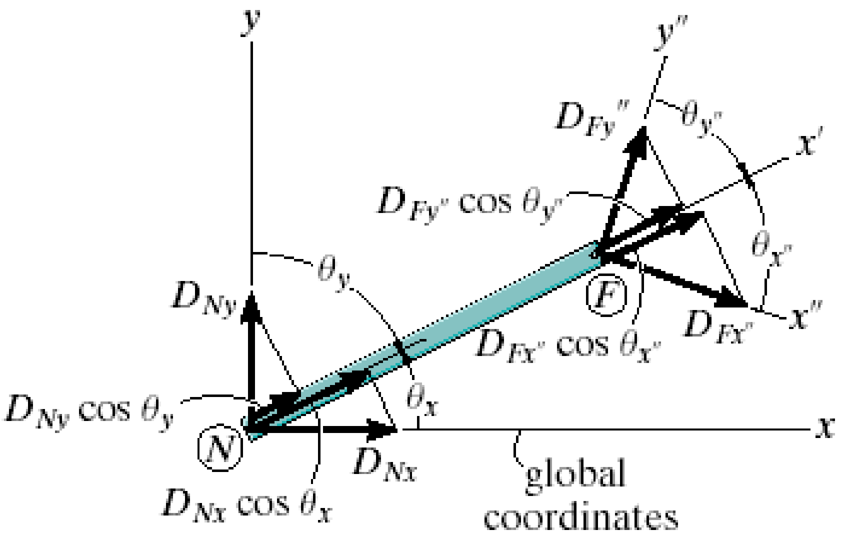
Nodal Coordinates

- Consider truss member 1 having a global coordinate system x, y at the near node and a nodal coordinate system x”, y” at the far node



Nodal Coordinates

- When displacement D occur so that they have components along each of these axes as shown



$$d_N = D_{N_x}\cos\theta_x + D_{N_y}\cos\theta_y$$

$$d_F = D_{F_x}\cos\theta_{x''} + D_{F_y}\cos\theta_{y''}$$

$$\begin{bmatrix} d_N \\ d_F \end{bmatrix} = \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_{x''} & \lambda_{y''} \end{bmatrix} \begin{bmatrix} D_{N_x} \\ D_{N_y} \\ D_{F_{x''}} \\ D_{F_{y''}} \end{bmatrix}$$

Nodal Coordinates

- If q is applied to the bar, the global force components at Q are:

$$Q_{N_x} = q_N \cos\theta_x \qquad Q_{N_y} = q_N \cos\theta_y$$

$$Q_{F_{x''}} = q_F \cos\theta_{x''} \qquad Q_{F_{y''}} = q_F \cos\theta_{y''}$$

- This can be expressed as:

$$\begin{bmatrix} Q_{N_x} \\ Q_{N_y} \\ Q_{F_{x''}} \\ Q_{F_{y''}} \end{bmatrix} = \begin{bmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ 0 & \lambda_{x''} \\ 0 & \lambda_{y''} \end{bmatrix} \begin{bmatrix} q_N \\ q_F \end{bmatrix}$$

Nodal Coordinates

$$Q = T^T k' T D \Leftrightarrow Q = kD$$

- We have

$$k = T^T k' T$$

$$k = \begin{bmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ 0 & \lambda_{x''} \\ 0 & \lambda_{y''} \end{bmatrix} \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_x & \lambda_y & 0 & 0 \\ 0 & 0 & \lambda_{x''} & \lambda_{y''} \end{bmatrix}$$

Nodal Coordinates

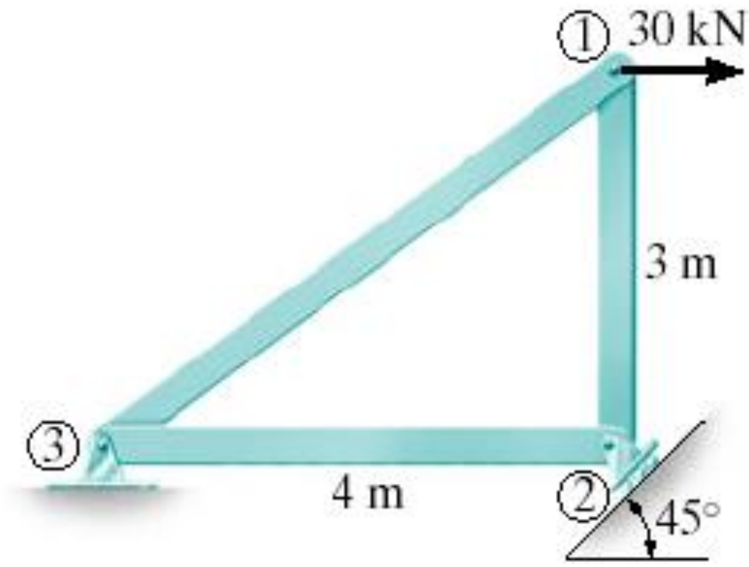
- Performing the matrix operation yields:

$$\mathbf{k} = \frac{AE}{L}\begin{bmatrix} \lambda_x^2 & \lambda_x\lambda_y & -\lambda_x\lambda_{x''} & -\lambda_x\lambda_{y''} \\ \lambda_x\lambda_y & \lambda_y^2 & -\lambda_y\lambda_{x''} & -\lambda_y\lambda_{y''} \\ -\lambda_x\lambda_{x''} & -\lambda_y\lambda_{x''} & \lambda_{x''}^2 & \lambda_{x''}\lambda_{y''} \\ -\lambda_x\lambda_{y''} & -\lambda_y\lambda_{y''} & \lambda_{x''}\lambda_{y''} & \lambda_{y''}^2 \end{bmatrix}$$

- This stiffness matrix is used for each member that is connected to an inclined roller support
- The process of assembling the matrices to form the structure stiffness matrix follows the standard procedure

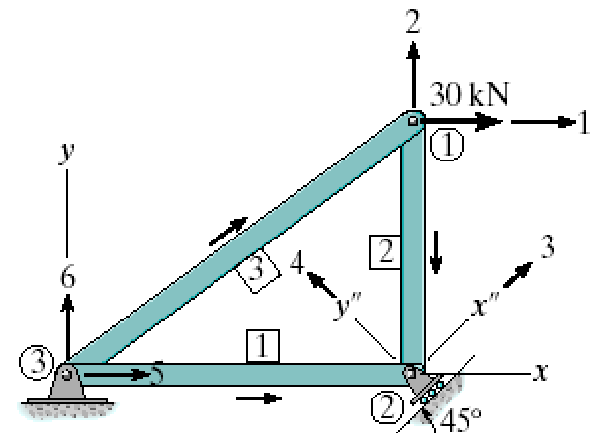
Example 2

Determine the support reactions for the truss as shown.

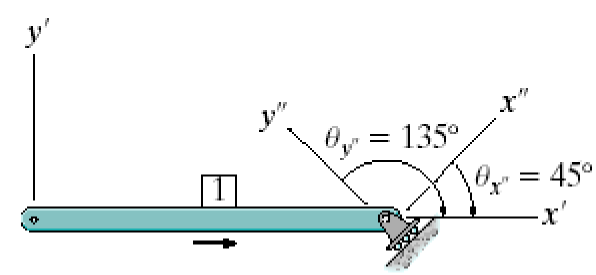


Solution

- Since the roller support at 2 is on an incline, we must use nodal coordinates at this node.
- The stiffness matrices for members 1 and 2 must be developed.
- Member 1,



(b)



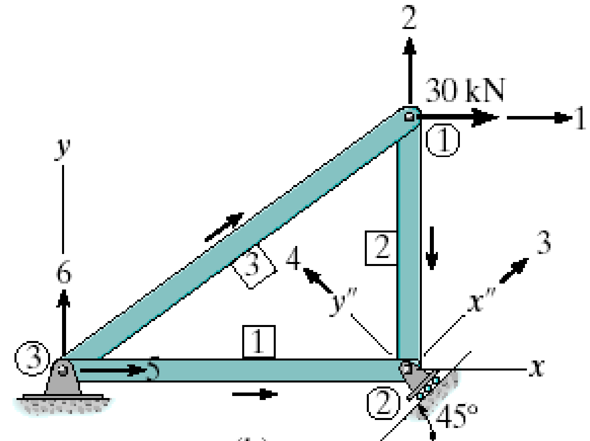
(c)

Solution

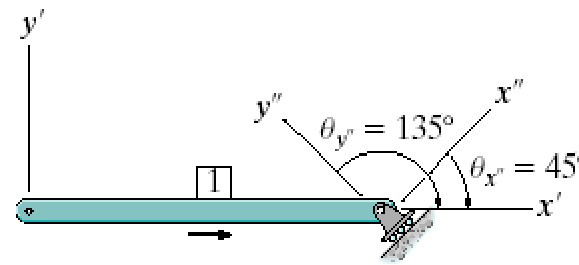
- Since the roller support at 2 is on an incline, we must use nodal coordinates at this node.
- The stiffness matrices for members 1 and 2 must be developed.
- Member 1,

$$\lambda_x = 1,\ \lambda_y = 0,\ \lambda_{x''} = 0.707,\ \lambda_{y''} = -0.707$$

$$\mathbf{k}_1 = AE \begin{array}{c} \begin{array}{cccc} 5 & 6 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0.25 & 0 & -0.17675 & 0.17675 \\ 0 & 0 & 0 & 0 \\ -0.17675 & 0 & 0.125 & -0.125 \\ 0.17675 & 0 & -0.125 & 0.125 \end{array}\right] \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 3 \\ 4 \end{array}$$



(b)



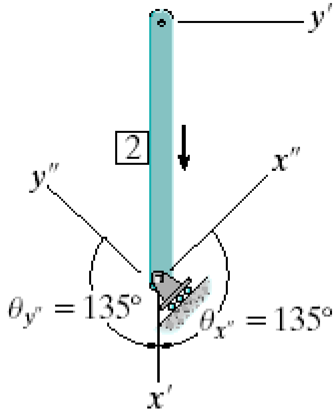
(c)

Solution

- Member 2,

$$\lambda_x = 0,\ \lambda_y = -1,\ \lambda_{x''} = -0.707,\ \lambda_{y''} = -0.707$$

$$\mathbf{k}_2 = AE \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0.3333 & -0.2357 & -0.2357 \\ 0 & -0.2357 & 0.1667 & 0.1667 \\ 0 & -0.2357 & 0.1667 & 0.1667 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$



Solution

- Member 3,

$$\lambda_x = 0.8,\ \lambda_y = 0.6$$

$$\mathbf{k}_3 = AE \begin{array}{c} \begin{array}{cccc} 5 & 6 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.128 & 0.096 & -0.128 & -0.096 \\ 0.096 & 0.072 & -0.096 & -0.072 \\ -0.128 & -0.096 & 0.128 & 0.096 \\ -0.096 & -0.072 & 0.096 & 0.072 \end{bmatrix} \end{array} \begin{array}{c} 5 \\ 6 \\ 1 \\ 2 \end{array}$$

Solution

- Assembling these matrices to determine the structure stiffness matrix, we have:

$$\begin{bmatrix} 30 \\ 0 \\ 0 \\ \hdashline Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = AE \left[\begin{array}{ccc:ccc} 0.128 & 0.096 & 0 & 0 & -0.128 & -0.096 \\ 0.096 & 0.4053 & -0.2357 & -0.2357 & -0.096 & -0.072 \\ 0 & -0.2357 & 0.2917 & 0.0417 & -0.17675 & 0 \\ \hdashline 0 & -0.2357 & 0.0417 & 0.2917 & 0.17675 & 0 \\ -0.128 & -0.096 & -0.17675 & 0.17675 & 0.378 & 0.096 \\ -0.096 & -0.072 & 0 & 0 & 0.096 & 0.072 \end{array}\right] \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ \hdashline 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution

- Carrying out the matrix multiplication of the upper partitioned matrices, the three unknown displacement D are determined from solving the resulting simultaneous equation.

$$D_1 = \frac{352.5}{AE} \quad , \quad D_2 = \frac{-157.5}{AE} \quad , \quad D_3 = \frac{-127.3}{AE}$$

- The unknown reactions Q are obtained from the multiplication of the lower partitioned matrices.

$$Q_4 = 31.8kN \quad , \quad Q_5 = -7.5kN \quad , \quad Q_6 = -22.5kN$$

Trusses having thermal changes & fabrication errors

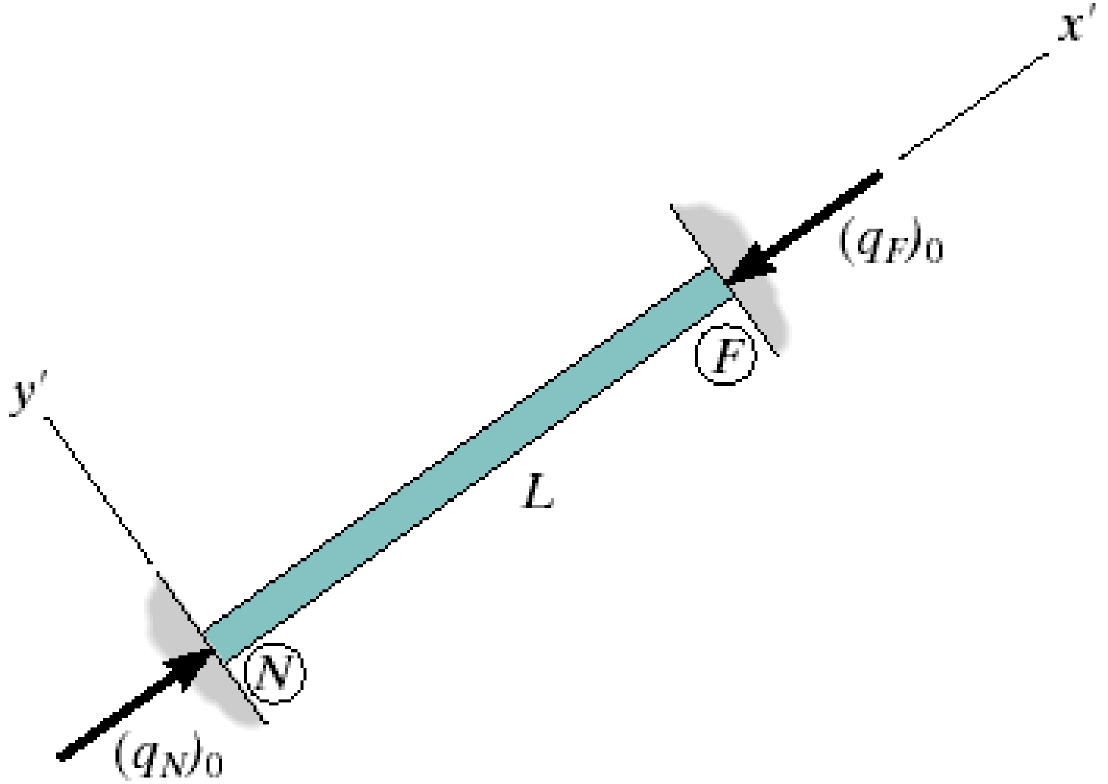
- If some of the members of the truss are subjected to an increase or decrease in length due to thermal changes or fabrication errors, then it is necessary to use the method of superposition to obtain the solution
- This requires 3 steps

Trusses having thermal changes & fabrication errors

- First, the fixed end forces necessary to prevent movement of the nodes as caused by temperature or fabrication are calculated
- Second, equal but opposite forces are placed on the truss at the nodes & the displacement of the nodes are calculated using the matrix analysis
- Third, the actual forces in the members & the reactions on the truss are determined by superposing these 2 results

Trusses having thermal changes & fabrication errors

- This force will hold the nodes of the member fixed as shown



Trusses having thermal changes & fabrication errors

- This procedure is only necessary if the truss is statically indeterminate
- If a truss member of length L is subjected to a temperature increase ΔT, the member will undergo an increase in length of $\Delta L = \alpha \, \Delta T L$
- A compressive force q0 applied to the member will cause a decrease in the member's length of $\Delta L' = q0L/AE$
- If we equate these 2 disp $q0 = AE\alpha \, \Delta T$

Trusses having thermal changes & fabrication errors

- This force will hold the nodes of the member fixed as shown in the figure

$$(q_N)_0 = AE\alpha\Delta T$$

$$(q_F)_0 = -AE\alpha\Delta T$$

- If a temperature decrease occurs then ΔT becomes negative & these forces reverse direction to hold the member in eqm

$$\begin{bmatrix} (Q_{Nx})_0 \\ (Q_{Ny})_0 \\ (Q_{Fx})_0 \\ (Q_{Fy})_0 \end{bmatrix} = \begin{bmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ 0 & \lambda_x \\ 0 & \lambda_y \end{bmatrix} AE\alpha\Delta T \begin{bmatrix} 1 \\ -1 \end{bmatrix} = AE\alpha\Delta T \begin{bmatrix} \lambda_x \\ \lambda_y \\ -\lambda_x \\ -\lambda_y \end{bmatrix}$$

Trusses having thermal changes & fabrication errors

- If a truss member is made too long by an amount ΔL before it is fitted into a truss, the force q0 needed to keep the member at its design length L is q0 = AEΔL /L

$$(q_N)_0 = \frac{AE\Delta L}{L}$$

$$(q_F)_0 = -\frac{AE\Delta L}{L}$$

Trusses having thermal changes & fabrication errors

- If the member is too short, then ΔL becomes negative & these forces will reverse
- In global coordinates, these forces are:

$$\begin{bmatrix} (Q_{Nx})_0 \\ (Q_{Ny})_0 \\ (Q_{Fx})_0 \\ (Q_{Fy})_0 \end{bmatrix} = \frac{AE\Delta L}{L} \begin{bmatrix} \lambda_x \\ \lambda_y \\ -\lambda_x \\ -\lambda_y \end{bmatrix}$$

Trusses having thermal changes & fabrication errors

- With the truss subjected to applied forces, temperature changes and fabrication errors, the initial force-displacement relationship for the truss then becomes:

$$Q = KD + Q_0$$

- Qo is the column matrix for the entire truss of the initial fixed-end forces caused by temperature changes & fabrication errors

$$\begin{bmatrix} \mathbf{Q}_k \\ \hdashline \mathbf{Q}_u \end{bmatrix} = \left[\begin{array}{c:c} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \hdashline \mathbf{K}_{21} & \mathbf{K}_{22} \end{array}\right] \begin{bmatrix} \mathbf{D}_u \\ \hdashline \mathbf{D}_k \end{bmatrix} + \begin{bmatrix} (\mathbf{Q}_k)_0 \\ \hdashline (\mathbf{Q}_u)_0 \end{bmatrix}$$

Trusses having thermal changes & fabrication errors

- Carrying out the multiplication, we obtain:

$$Q_k = K_{11}D_u + K_{12}D_k + (Q_k)_0$$
$$Q_u = K_{21}D_u + K_{22}D_k + (Q_u)_0$$

- According to the superposition procedure described above, the unknown disp are determined from the first eqn by subtracting K12Dk and (Qk)0 from both sides & then solving for Du

Trusses having thermal changes & fabrication errors

- Once these nodal displacement are obtained, the member forces are determined by superposition:

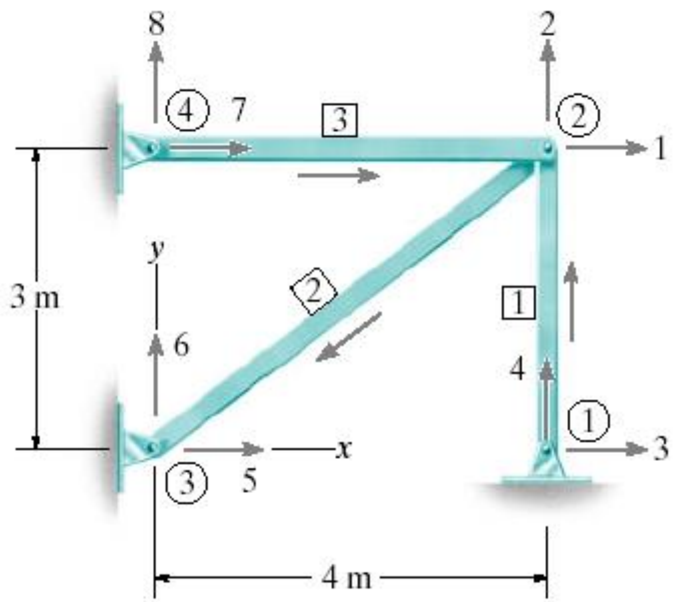
$$q = k'TD + q_0$$

- If this equation is expanded to determine the force at the far end of the member, we obtain:

$$q_F = \frac{AE}{L}\left[-\lambda_x - \lambda_y \; \lambda_x \; \lambda_y\right]\begin{bmatrix} D_{Nx} \\ D_{Ny} \\ D_{Fx} \\ D_{Fy} \end{bmatrix} - (q_F)_0$$

Example 3

Determine the force in member 1 & 2 of the pin-connected assembly if member 2 was made 0.01 m too short before it was fitted into place. Take AE = $8(10^3)$kN.



Example 3

- Since the member is short, then ΔL = -0.01m.
- For member 2, we have

$$\begin{bmatrix} (Q_1)_0 \\ (Q_2)_0 \\ (Q_5)_0 \\ (Q_6)_0 \end{bmatrix} = \frac{AE(-0.01)}{5} \begin{bmatrix} -0.8 \\ -0.6 \\ 0.8 \\ 0.6 \end{bmatrix} = AE \begin{array}{c} \begin{bmatrix} 0.0016 \\ 0.0012 \\ -0.0016 \\ -0.0012 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 5 \\ 6 \end{array}$$

Example 3

- Assembling the stiffness matrix, we have

$$\begin{bmatrix} 0 \\ 0 \\ \hdashline Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix} = AE \left[\begin{array}{cc:cccccc} 0.378 & 0.096 & 0 & 0 & -0.128 & -0.096 & -0.25 & 0 \\ 0.096 & 0.405 & 0 & -0.333 & -0.096 & -0.072 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.333 & 0 & 0.333 & 0 & 0 & 0 & 0 \\ -0.128 & -0.096 & 0 & 0 & 0.128 & 0.096 & 0 & 0 \\ -0.096 & -0.072 & 0 & 0 & 0.096 & 0.072 & 0 & 0 \\ -0.25 & 0 & 0 & 0 & 0 & 0 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} D_1 \\ D_2 \\ \hdashline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + AE \begin{bmatrix} 0.0016 \\ 0.0012 \\ \hdashline 0 \\ 0 \\ -0.0016 \\ -0.0012 \\ 0 \\ 0 \end{bmatrix}$$

Example 3

- Partitioning the matrices as shown & carrying out the multiplication to obtain the eqn for the unknown disp yields.

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = AE \begin{bmatrix} 0.378 & 0.096 \\ 0.096 & 0.405 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} + AE \begin{bmatrix} 0 & 0 & -0.128 & -0.096 & -0.25 & 0 \\ 0 & -0.333 & -0.096 & -0.072 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + AE \begin{bmatrix} 0.0016 \\ 0.0012 \end{bmatrix}$$

- Solving simultaneous eqation gives:

Example 3

- Member 1

$$\lambda_x = 0,\ \lambda_y = 1,\ L = 3\text{m, AE} = 8(10^3)kN$$

$$q_1 = -5.56kN$$

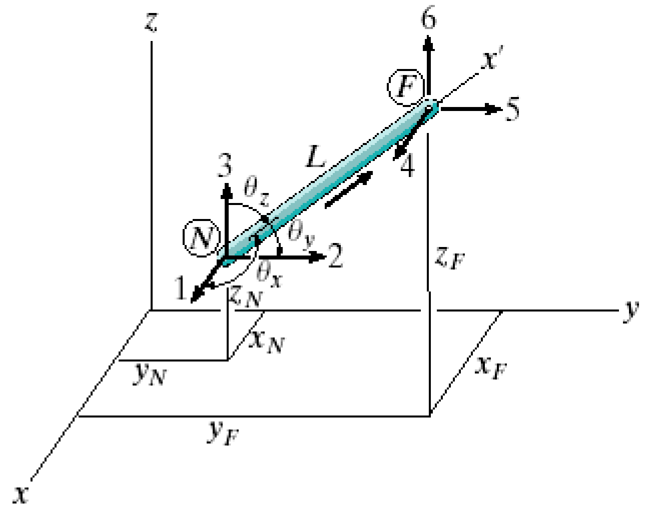
- Member 2

$$\lambda_x = -0.8,\ \lambda_y = -0.6,\ L = 5\text{m, AE} = 8(10^3)kN$$

$$q_2 = 9.26kN$$

Space-truss analysis

- The analysis of both statically determinate and indeterminate space trusses can be performed by using the same procedure discussed previously
- To account for the 3-D aspects of the problem, additional elements must be included in the transformation matrix T
- Consider the truss member



Space-truss analysis

- By inspection the direction cosines bet the global & local coordinates can be found

$$\lambda_x = \cos\theta_x = \frac{x_F - x_N}{L}$$

$$= \frac{x_F - x_N}{\sqrt{(x_F - x_N)^2 + (y_F - y_N)^2 + (z_F - z_N)^2}}$$

Space-truss analysis

$$\lambda_y = \cos\theta_y = \frac{y_F - y_N}{L}$$

$$= \frac{y_F - y_N}{\sqrt{(x_F - x_N)^2 + (y_F - y_N)^2 + (z_F - z_N)^2}}$$

$$\lambda_z = \cos\theta_z = \frac{z_F - z_N}{L}$$

$$= \frac{z_F - z_N}{\sqrt{(x_F - x_N)^2 + (y_F - y_N)^2 + (z_F - z_N)^2}}$$

Space-truss analysis

- As a result of the third dimension, the transformation matrix becomes:

$$\mathbf{T} = \begin{bmatrix} \lambda_x & \lambda_y & \lambda_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_x & \lambda_y & \lambda_z \end{bmatrix}$$

- By substitution, we have

$$\mathbf{k} = \begin{bmatrix} \lambda_x & 0 \\ \lambda_y & 0 \\ \lambda_z & 0 \\ 0 & \lambda_x \\ 0 & \lambda_y \\ 0 & \lambda_z \end{bmatrix} \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_x & \lambda_y & \lambda_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_x & \lambda_y & \lambda_z \end{bmatrix}$$

Space-truss analysis

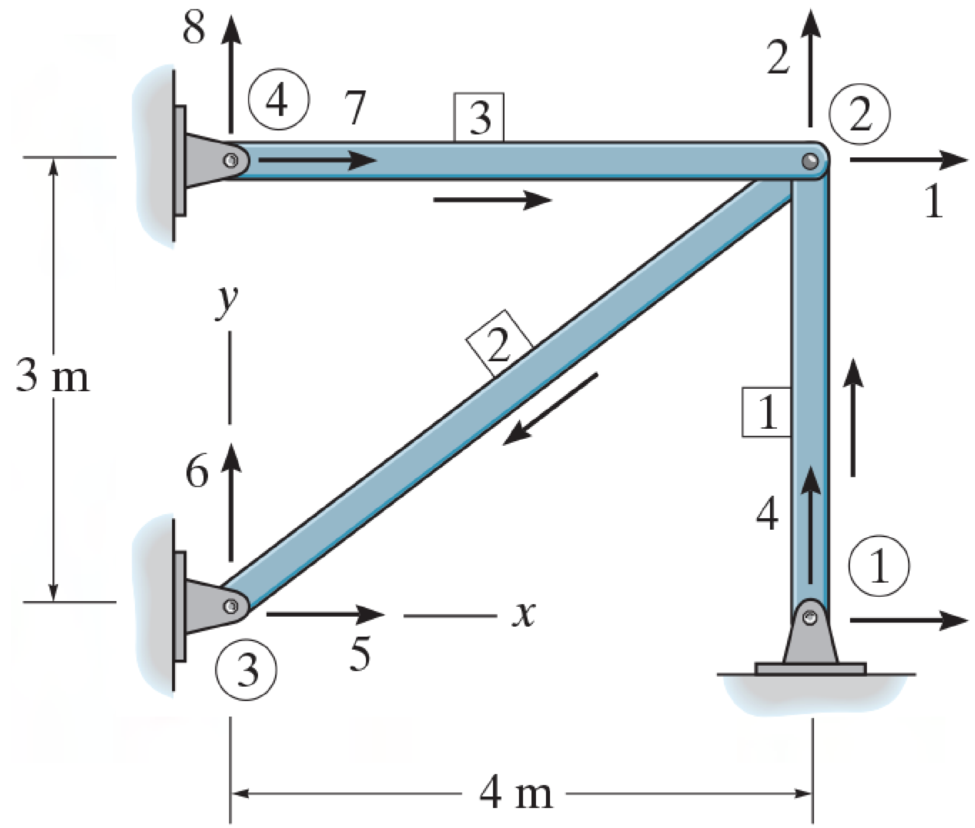
- Carrying out the matrix multiplication yields the symmetric matrix

$$
\mathbf{k} = \frac{AE}{L}
\begin{array}{c}
\begin{array}{cccccc} N_x & N_y & N_z & F_x & F_y & F_z \end{array} \\
\left[\begin{array}{cccccc}
\lambda_x^2 & \lambda_x\lambda_y & \lambda_x\lambda_z & -\lambda_x^2 & -\lambda_x\lambda_y & -\lambda_x\lambda_z \\
\lambda_y\lambda_x & \lambda_y^2 & \lambda_y\lambda_z & -\lambda_y\lambda_x & -\lambda_y^2 & -\lambda_y\lambda_z \\
\lambda_z\lambda_x & \lambda_z\lambda_y & \lambda_z^2 & -\lambda_z\lambda_x & -\lambda_z\lambda_y & -\lambda_z^2 \\
-\lambda_x^2 & -\lambda_x\lambda_y & -\lambda_x\lambda_z & \lambda_x^2 & \lambda_x\lambda_y & \lambda_x\lambda_z \\
-\lambda_y\lambda_x & -\lambda_y^2 & -\lambda_y\lambda_z & \lambda_y\lambda_x & \lambda_y^2 & \lambda_y\lambda_z \\
-\lambda_z\lambda_x & -\lambda_z\lambda_y & -\lambda_z^2 & \lambda_z\lambda_x & \lambda_z\lambda_y & \lambda_z^2
\end{array}\right]
\end{array}
\begin{array}{c} \\ N_x \\ N_y \\ N_z \\ F_x \\ F_y \\ F_z \end{array}
$$

- This equation represent the member stiffness matrix expressed in global coordinates

Example 4

Determine the force in each member of the 3-member truss as shown. AE is constant.



Example 5

Determine the force in each member of the 6-member truss as shown. AE is constant