Lecture 5

Virtual Work Method (1)

Mohamad Fathi GHANAMEH

WORK

Basic definition

Work is done when a force moves its point of application

WORK

Work done by a force
The body on which it acts is given a displacement, which is coincident with the line of action of the force.

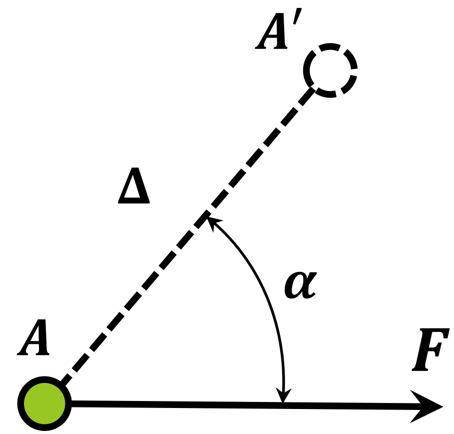
Work done by both forces and moments
The body on which it acts is given a displacement, which is not coincident with the line of action of the force.

More exact definition

WORK

Case 1

work done by a force, and the displacement is not coincident with the line of action of the force.



- F acting on a particle A
- The particle is given a displacement *Δ*
- The particle moves to A′ in a direction at an angle α to the line of action of F

$$W_F = F(\Delta . \cos\alpha) \quad \text{or} \quad W_F = (F . \cos\alpha)\Delta$$

Product of F and the component of *Δ* in the direction of F

Product of the component of F in the direction of *Δ* and *Δ*

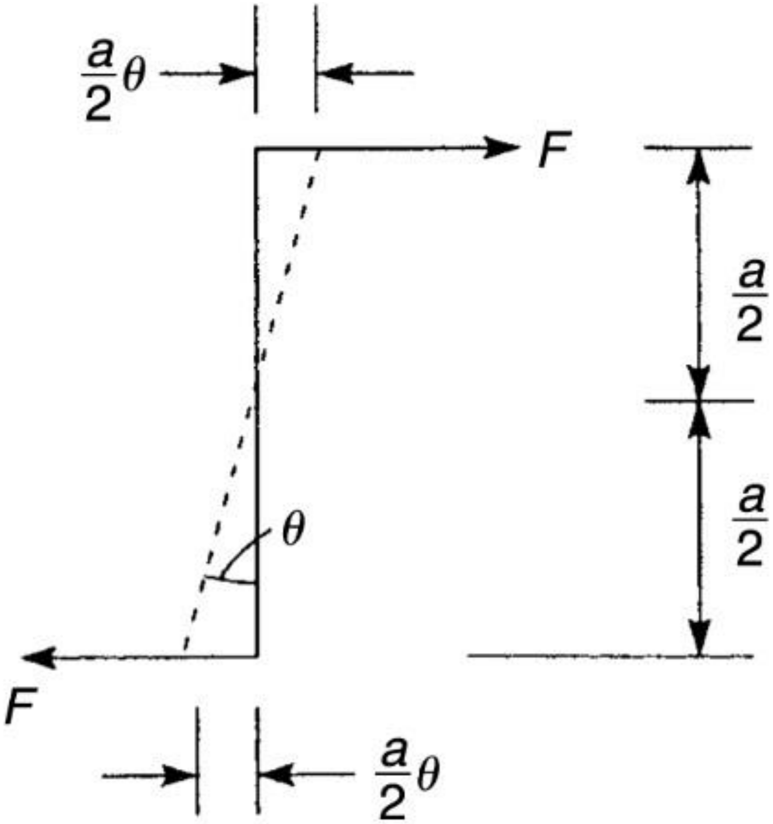
WORK

Case 2

work done by a couple

The couple is given a small rotation of θ radians.

$$W_C = F\frac{a}{2}\theta + F\frac{a}{2}\theta = F.a.\theta$$



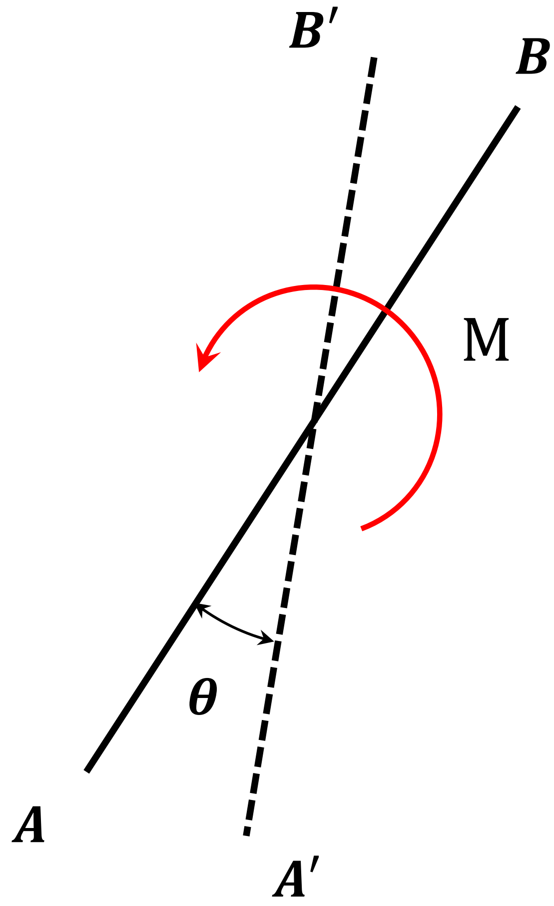
WORK

Case 3

Work done by pure moment

M acting on the bar AB in as it is given a small rotation, θ

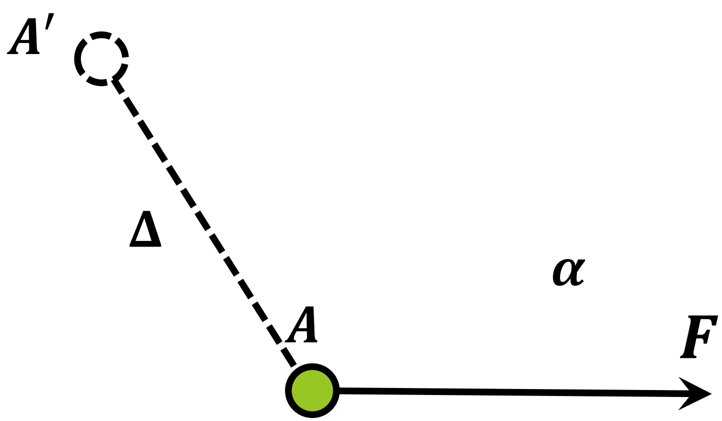
$$W_M = M\,.\theta$$



WORK

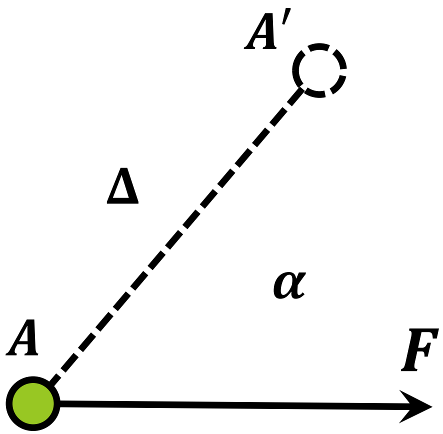
Work done is negative

The F and the component of Δ have opposite directions.



Work done is positive

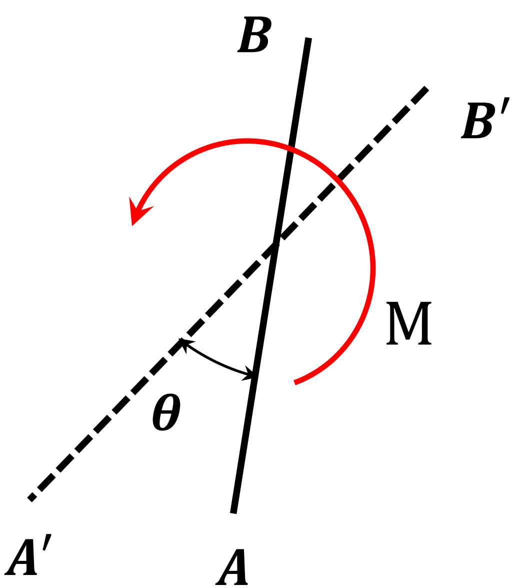
The F and the component of Δ have the same direction.



WORK

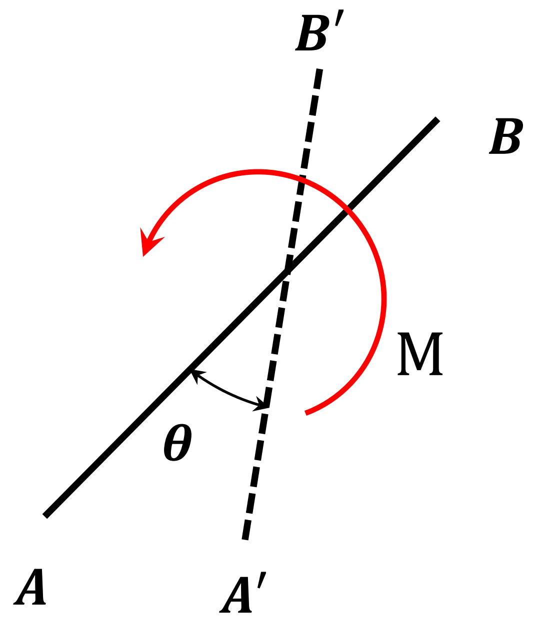
Work done is negative

The M and the rotation θ have opposite senses



Work done is positive

The M and the rotation θ have the same sense



WORK

The displacement, Δ, had been perpendicular to the force, F , no work would have been done by F.

The work is a scalar quantity since it is not associated with direction (the force F does work if the particle is moved in any direction).

The work done by a series of forces is the algebraic sum of the work done by each force.

Virtual Work for a Particle

A particle A is acted on by a number of concurrent forces:

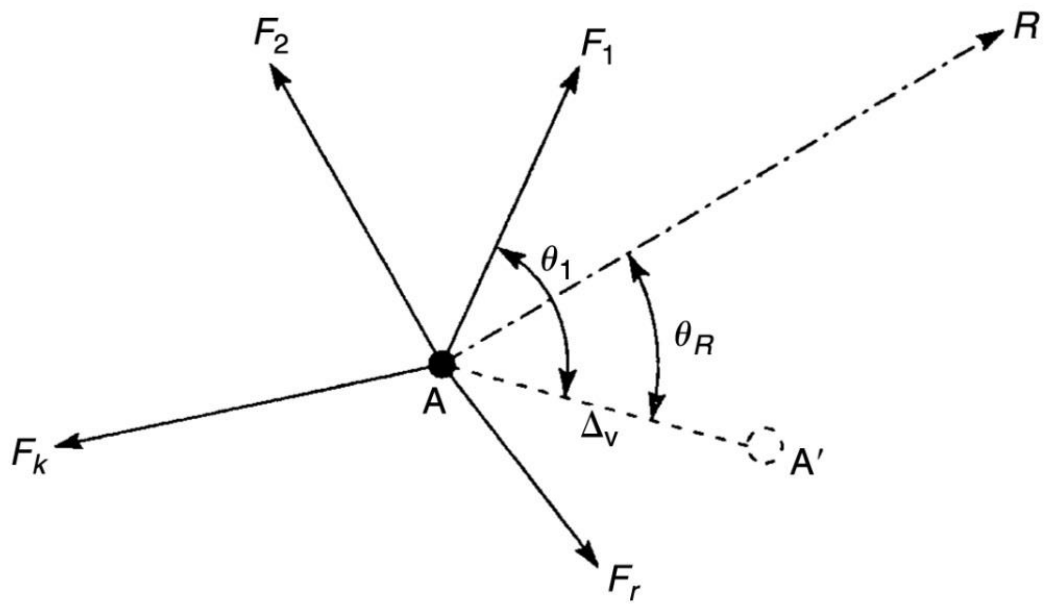
$$F_1, F_2, \ldots, F_k, \ldots, F_r$$

If it is given a small arbitrary displacement, Δ_V , to A′

Δ_V is sufficiently small so that the directions of the forces are unchanged.

The angles that the forces make with the direction of Δ_V

$$\theta_1, \theta_2, \ldots, \theta_k, \ldots, \theta_r$$



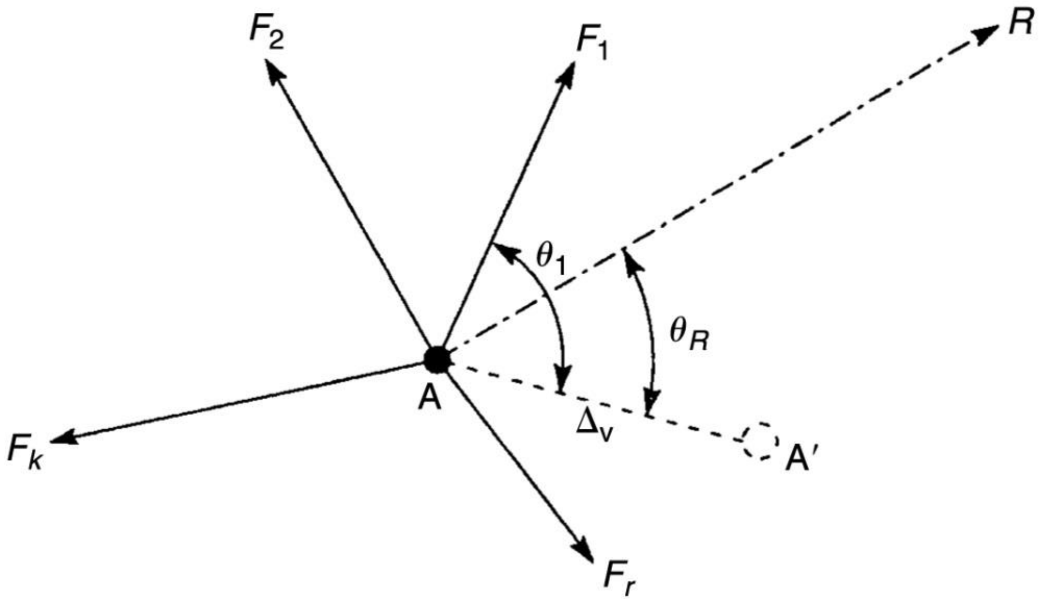
Virtual Work for a Particle

$$W_F = F_1.\Delta_v.cos\theta_1 + F_2.\Delta_v.cos\theta_2 + \ldots + F_k.\Delta_v.cos\theta_k + \ldots + F_r.\Delta_v.cos\theta_r$$

$$W_F = \sum_{k=1}^{r} F_k.\Delta_v.cos\theta_k$$

$$W_F = \Delta_v.\sum_{k=1}^{r} F_k.cos\theta_k$$

- The resultant of these forces is R.
- The angle that the resultant R makes with the direction of Δ_V, is θ_R



$$\sum_{k=1}^{r} F_k.cos\theta_k = R.cos\theta_R \Rightarrow W_F = \Delta_v.R.cos\theta_R$$

Virtual Work for a Particle

Principle of virtual work for a particle

If a particle is in equilibrium under the action of a number of forces, the total work done by the forces for a small arbitrary displacement of the particle is zero.

Total work done by the forces can be zero even though the particle is not in equilibrium if the virtual displacement is taken to be in a direction perpendicular to their resultant, R.

A particle is in equilibrium under the action of a system of forces if the total work done by the forces is zero for any virtual displacement of the particle.

Virtual Work for a Rigid Body

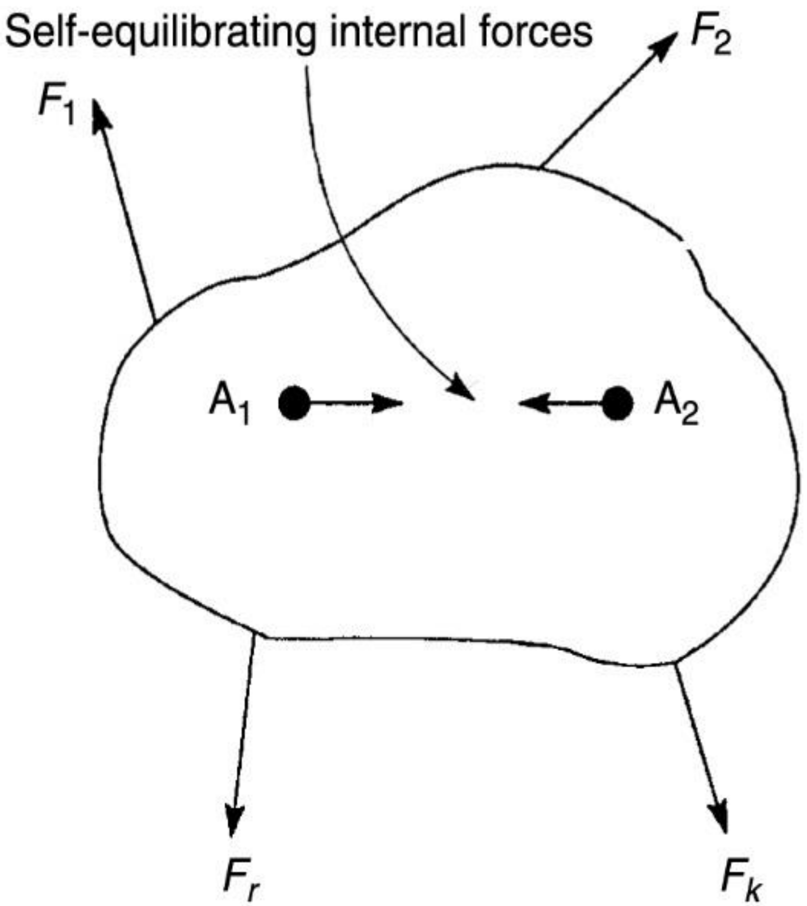
A rigid body is acted on by a system of external forces:

$$F_1, F_2, \ldots, F_k, \ldots, F_r$$

External forces induce internal forces

On adjacent particles, such as A_1 and A_2, these internal forces will be equal and opposite.

Suppose that the rigid body is given a small, imaginary, displacement or a rotation or a combination of both, in some specified direction.

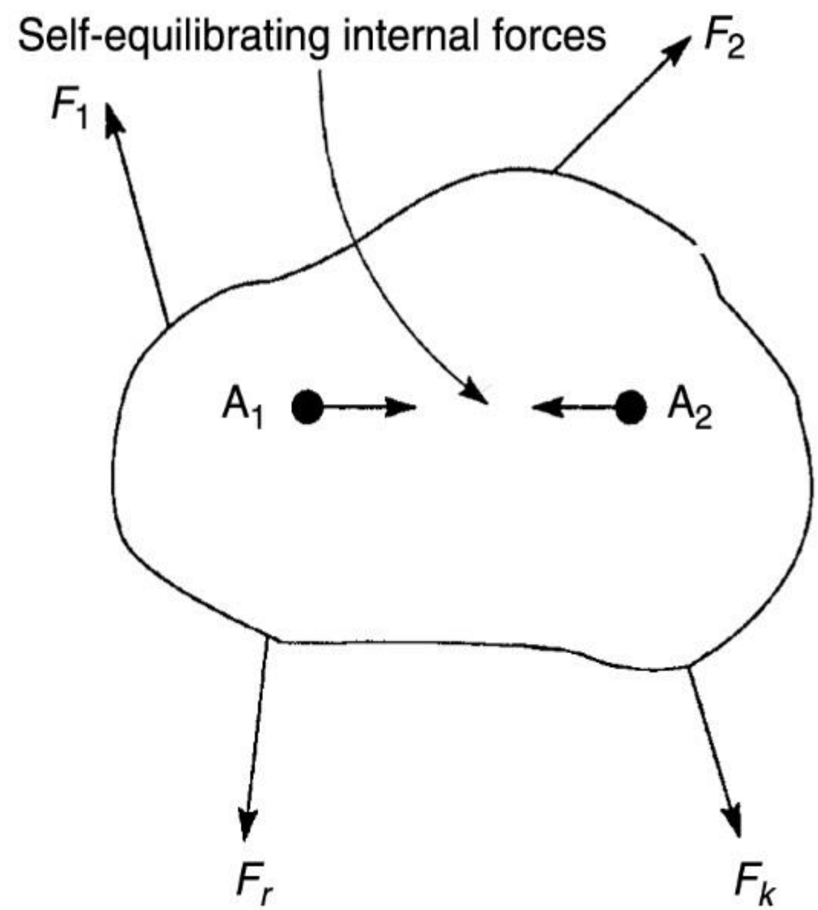


Virtual Work for a Rigid Body

The total virtual work done W_t is the sum of the virtual work W_e done by the external forces and the virtual work W_i done by the internal force

$$W_t = W_e + W_i$$

Since the body is rigid, all the particles in the body move through the same displacement so that the virtual work done on all the particles is numerically the same.

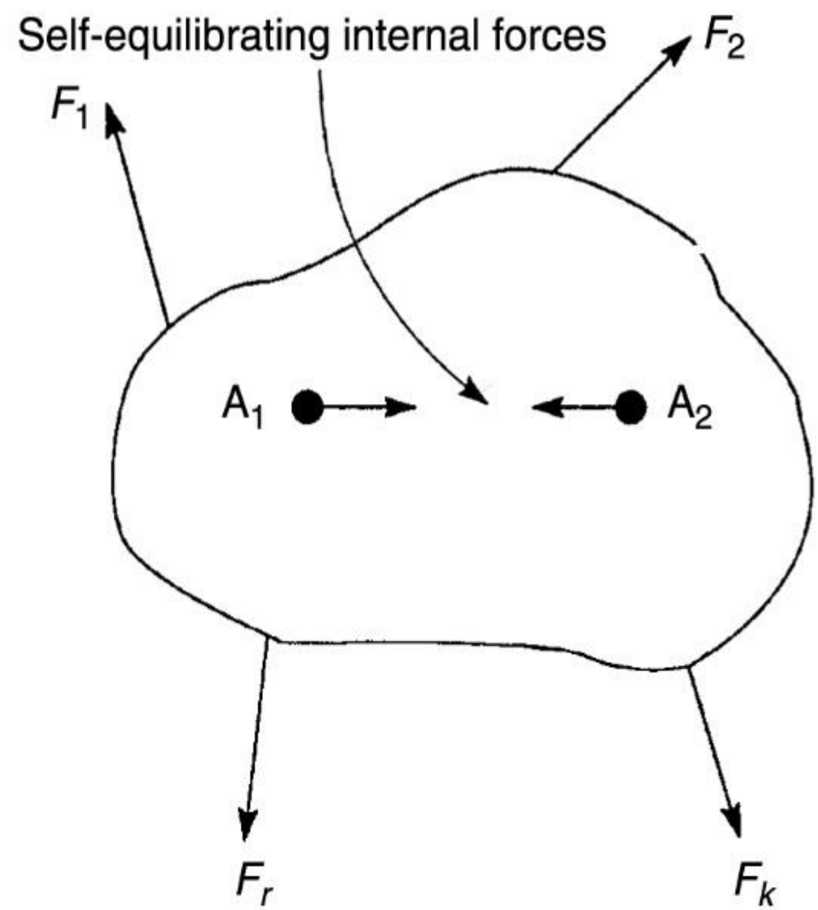


Virtual Work for a Rigid Body

for a pair of adjacent particles, such as A_1 and A_2, the self-equilibrating forces are in opposite directions, which means that the work done on A_1 is opposite in sign to the work done on A_2. Therefore, the sum of the virtual work done on A_1 and A_2 is zero

$$W_t = W_e$$

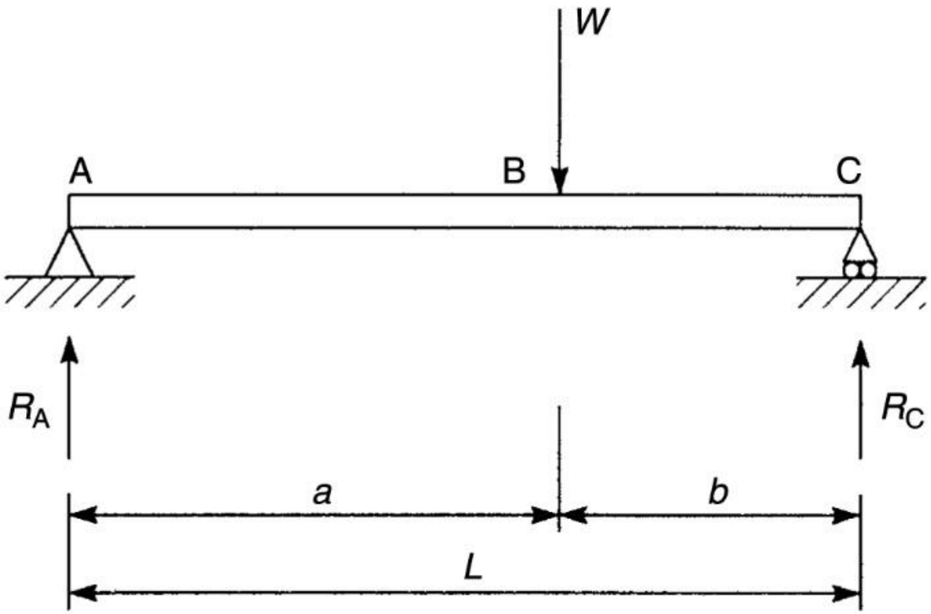
we may regard the body as a large particle.



Virtual Work for a Rigid Body

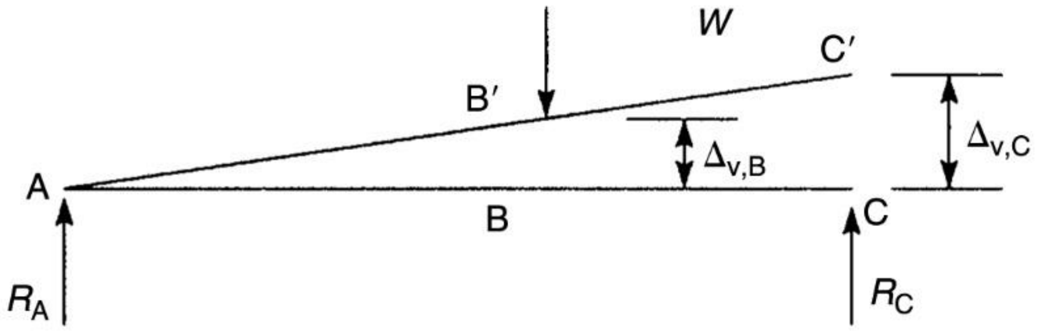
Example

Calculate the support reactions in the simply supported beam



Virtual Work for a Rigid Body

$$W_t = R_C \cdot \Delta_{v,\mathrm{C}} - W \cdot \Delta_{v,B}$$

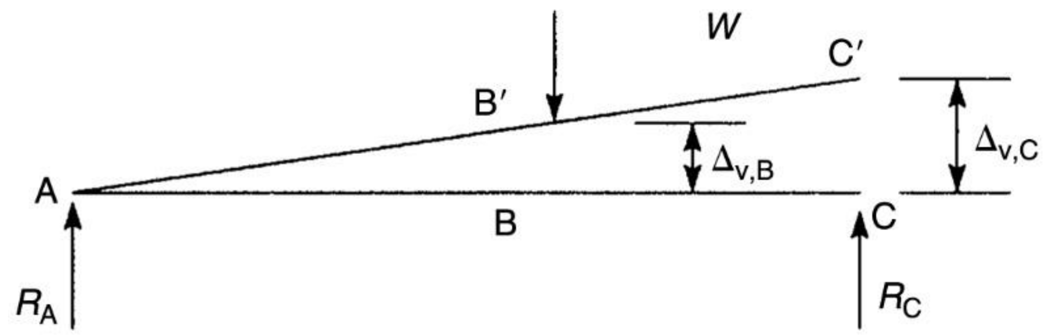


$$\Delta_{v,B} = \frac{a}{a+b}\Delta_{v,\mathrm{C}} = \frac{a}{L}\Delta_{v,\mathrm{C}}$$

$$W_t = R_C \cdot \Delta_{v,\mathrm{C}} - W\,\frac{a}{L}\Delta_{v,\mathrm{C}}$$

Principle of Virtual Work for a Rigid Body

Since the beam is in equilibrium



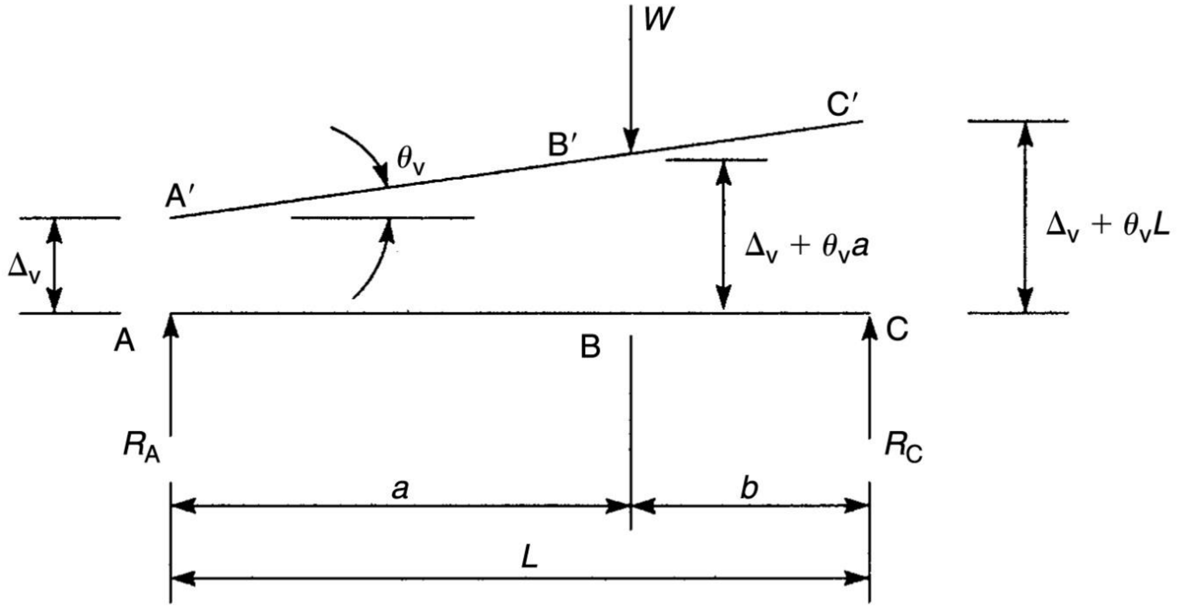
$$0 = R_C \cdot \Delta_{v,C} - W\frac{a}{L}\Delta_{v,C}$$

$$R_C = W\frac{a}{L}$$

Virtual Work for a Rigid Body

Suppose now that instead of the single displacement $\Delta_{v,C}$, the complete beam is given a vertical virtual displacement Δ_v together with a virtual rotation θ_v about A

The total virtual work W_t done by the forces acting on the beam is now given by:



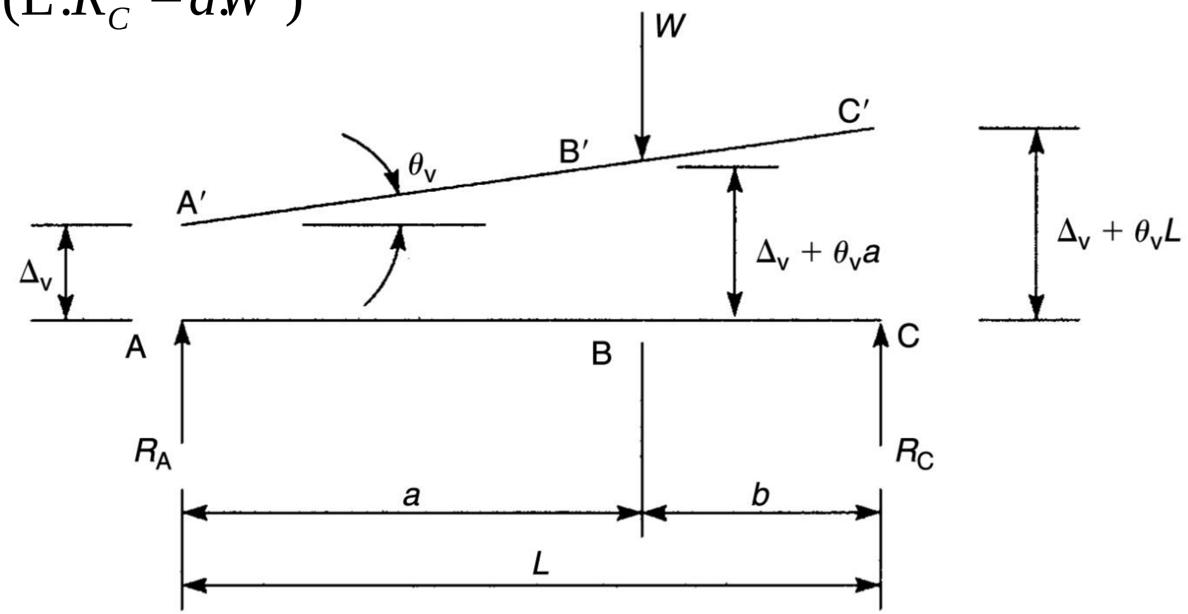
$$W_t = R_A.\Delta_v - W(\Delta_v + a.\theta_V) + R_C(\Delta_v + L.\theta_V)$$

$$W_t = \Delta_v(R_A - W + R_C) + \theta_V(\mathrm{L}.R_C - aW)$$

Virtual Work for a Rigid Body

Since the beam is in equilibrium

$$0 = \Delta_v (R_A - W + R_C) + \theta_V (\mathrm{L}.R_C - aW)$$



This equation is valid for all values of Δ_V and θ_V so that:

$$R_A - W + R_C = 0$$

$$\mathrm{L}.R_C - aW = 0$$