Aerospace Structural Analysis

Lecture 4

Theories of Failure

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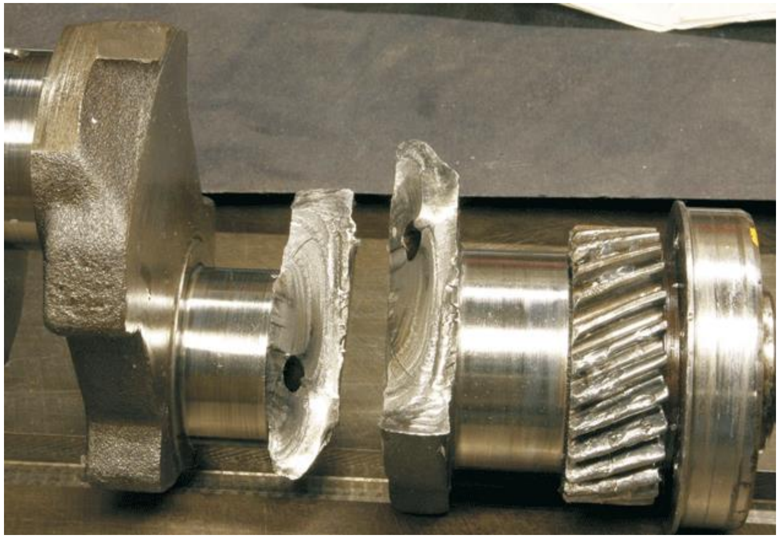
What is Failure?

What is Failure?

Failure : Any change in a machine part which makes it unable to perform its intended function.

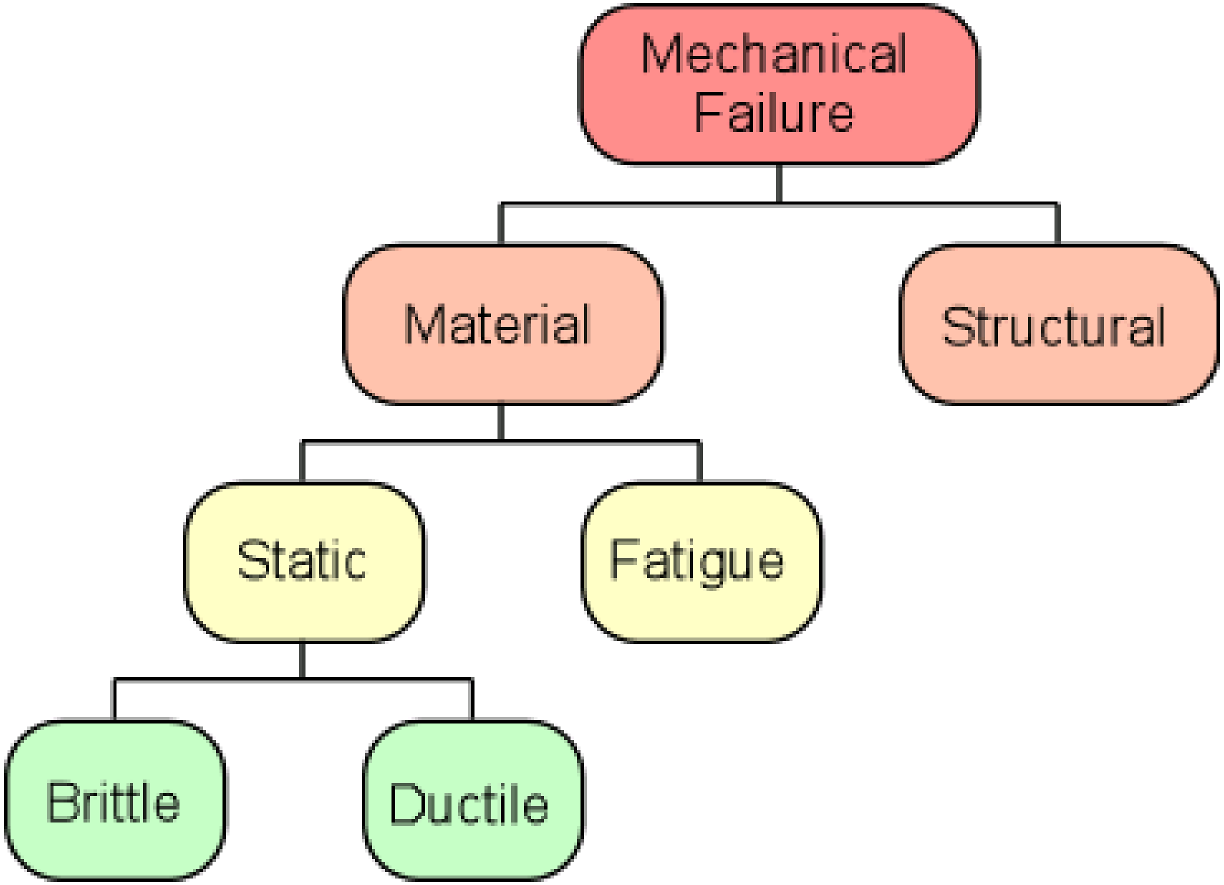


What is Failure?

Failure of a member is defined as one of two conditions:

1. Fracture of the material of which the member is made. This type of failure is the characteristic of brittle materials.
2. Initiation of inelastic (Plastic) behavior in the material. This type of failure is the one generally exhibited by ductile materials.

What is Failure?



Mechanical Failure

Material

Structural

Static

Fatigue

Brittle

Ductile

Ductile vs Brittle Failure

Ductile fracture

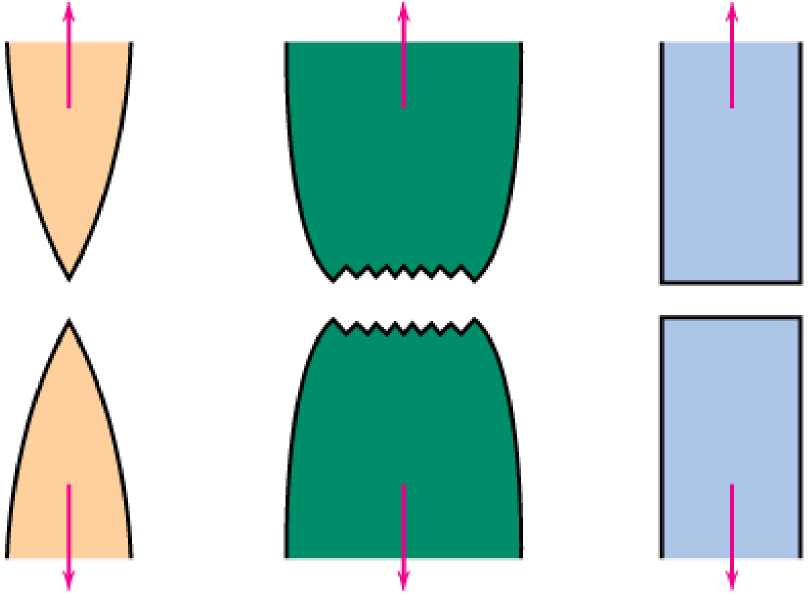
Occurs with plastic deformation

Brittle fracture

- **Occurs with Little or no plastic deformation**
- **Thus they are Catastrophic meaning they occur without warning!**

Ductile vs Brittle Failure

Fracture behavior:	Very Ductile	Moderately Ductile	Brittle



%*El*	Large	Moderate	Small

- Ductile fracture is nearly always desirable!

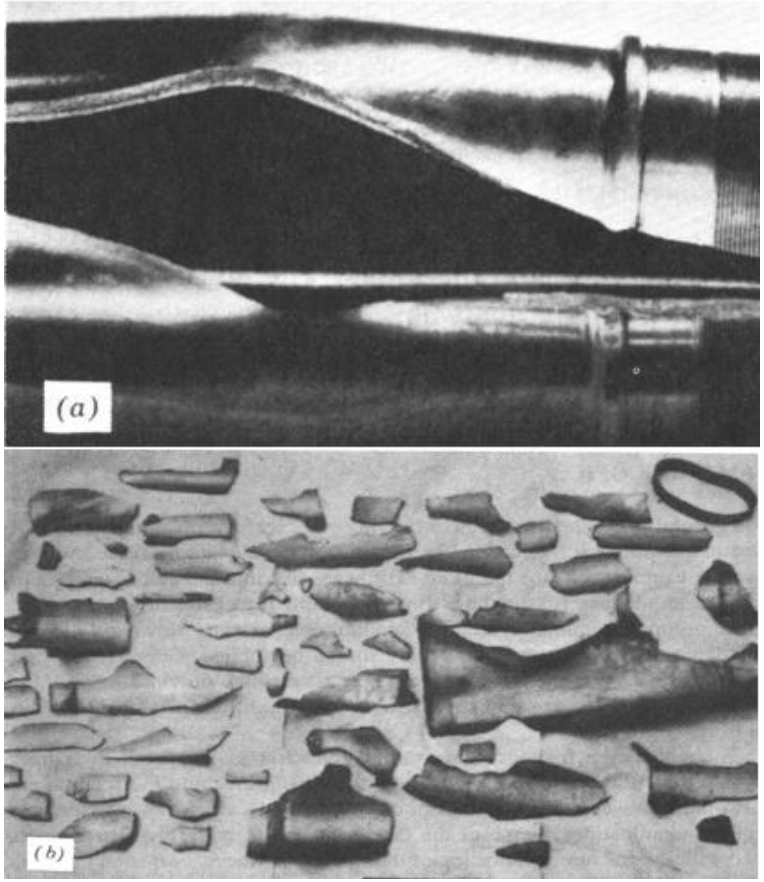
Ductile:	Brittle:
warning before fracture	No warning

Ductile vs Brittle Failure

Example: Failure of a Pipe

- **Ductile failure:**
  - --one piece
  - --large deformation

- **Brittle failure:**
  - --many pieces
  - --small deformation



V.J. Colangelo and F.A. Heiser, *Analysis of Metallurgical Failures* (2nd ed.)

Ductile vs Brittle Failure

cup-and-cone fracture

brittle fracture

Callister 7e.

Design and Failure

When an engineer is faced with the problem of design using a specific material

Member is subjected to a uniaxial state of stress

	material	upper limit
Failure	Ductile	Initiation of Yielding
	Brittle	Fracture

Member is subjected to biaxial or triaxle stress

Failure — Criterion for Failure

Theories of Failure

No single theory of failure, however, can be applied to a specific material at all times, because a material may behave in either a ductile or brittle manner depending on the temperature, rate of loading, chemical environment, or the way the material is shaped or formed.

Procedure for Failure Criterion

1. Calculate the normal and shear stress at points where they are the largest in the member.
2. Determine the principal stresses at these critical points.
3. Applying the appropriate theory of failure.

Theories of Failure

Ductile Materials

- Maximum-Shear-Stress Theory - Tresca yield criterion
- Maximum-Distortion-Energy Theory - Von Mises and H. Hencky

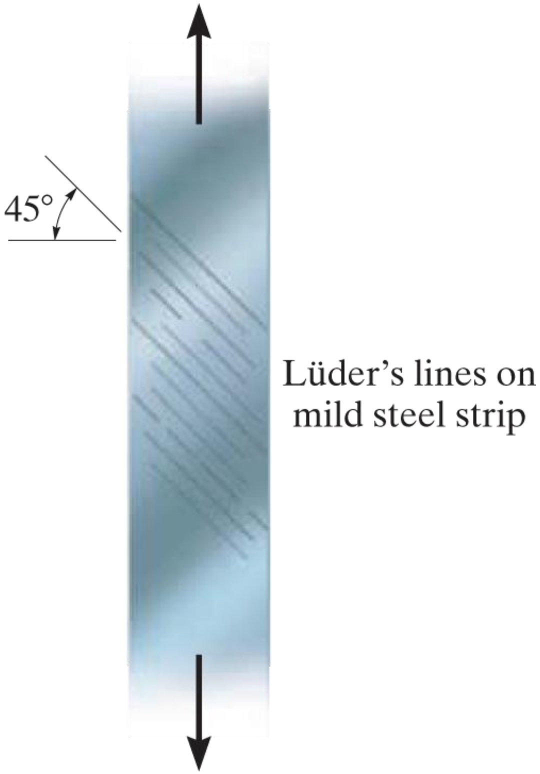
Brittle Materials

- Maximum-Normal-Stress Theory - W. Rankine
- Mohr’s Failure Criterion.

Maximum-Shear-Stress Theory

yielding of a ductile material

caused by slipping

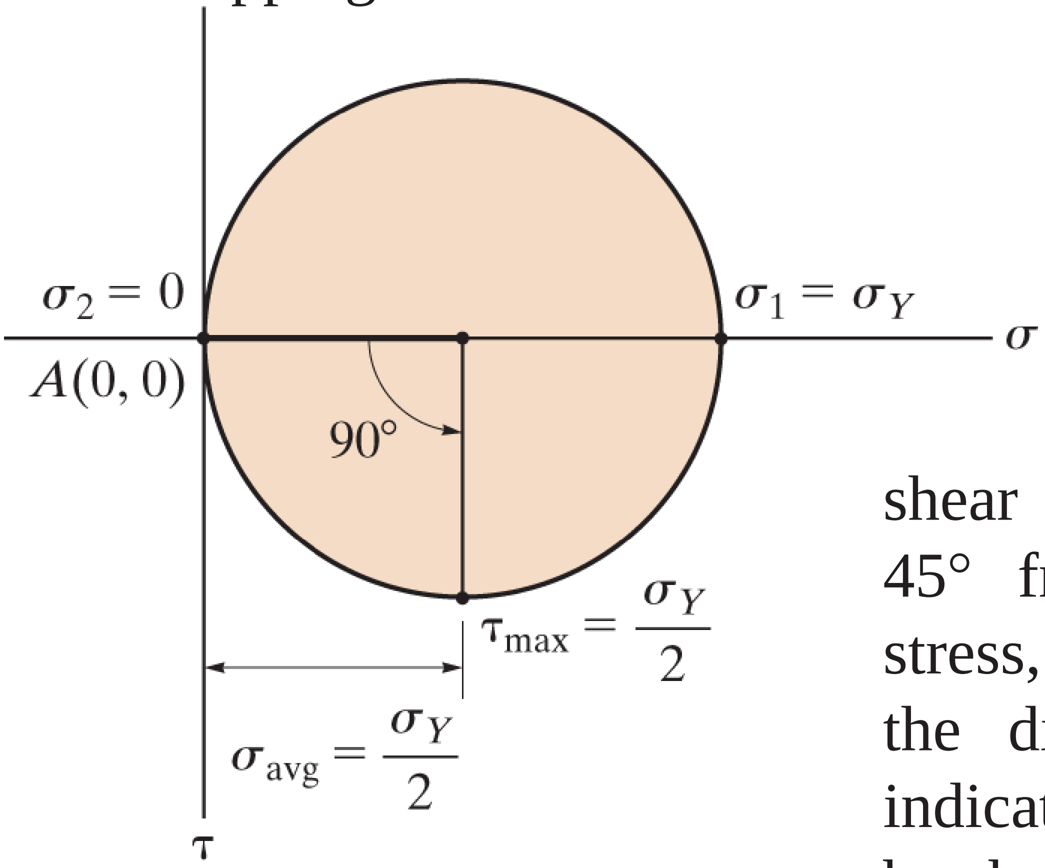


Specimen : a highly polished thin strip

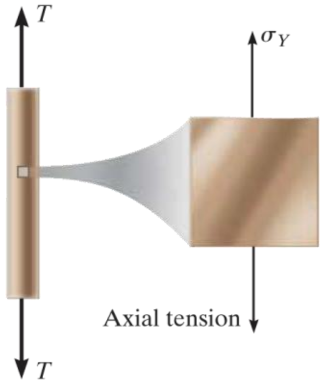
Subjected to : a simple tension test

Maximum-Shear-Stress Theory

The slipping that occurs is caused by shear stress.



Mohr's circle for the element



shear stress acts on planes that are 45° from the planes of principal stress, and these planes coincide with the direction of the Lüder lines, indicating that indeed failure occurs by shear.

Maximum-Shear-Stress Theory

1868 Henri Tresca proposed the **maximum-shear-stress theory** or **Tresca yield criterion**

Can be used to predict the failure stress of a ductile material subjected to any type of loading.

theory states that yielding of the material begins when the absolute maximum shear stress in the material reaches the shear stress that causes the same material to yield when it is subjected only to axial tension.

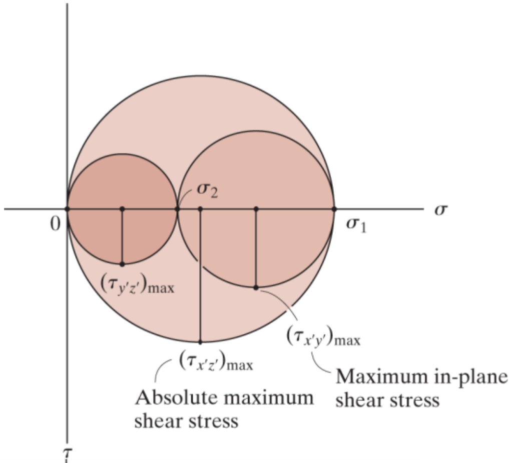
Therefore, to avoid failure, it is required that $\tau_{\substack{abs \\ max}}$ in the material must be less than or equal to $\frac{\sigma_Y}{2}$ where σ_Y is determined from a simple tension test.

Maximum-Shear-Stress Theory

Express the absolute maximum shear stress in terms of the principal stresses.

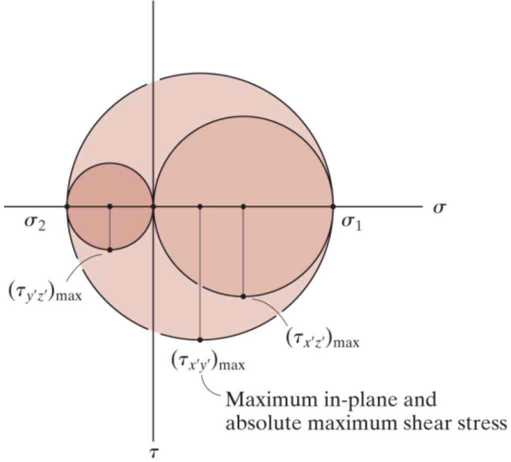
Both in-plane principal stresses have the *same sign*,
they are both tensile or both compressive.
failure will occur *out of the plane*

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_1}{2}$$



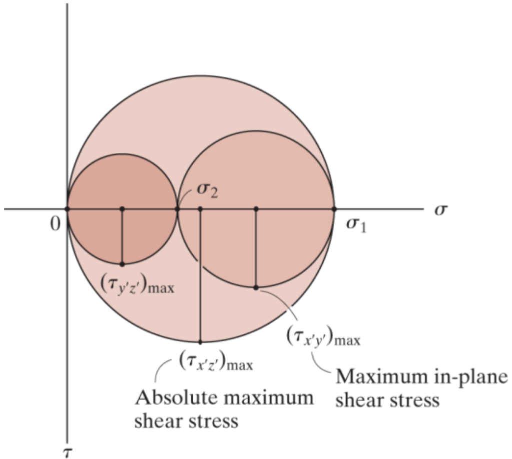
Both in-plane principal stresses have *opposite signs*,
one is tensile and the other is compressive.
failure will occur *in the plane*

$$\tau_{\substack{\text{abs}\\\text{max}}} = \frac{\sigma_1 - \sigma_2}{2}$$

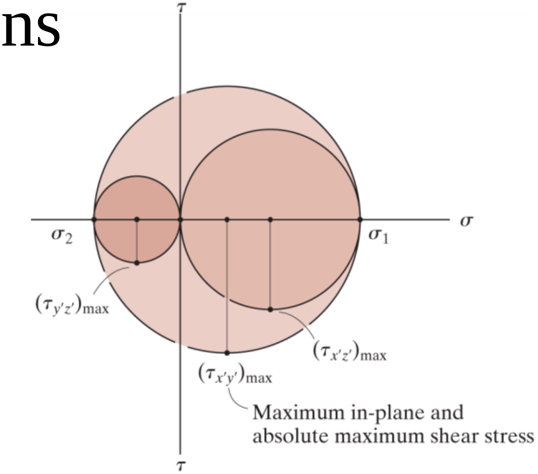


Maximum-Shear-Stress Theory

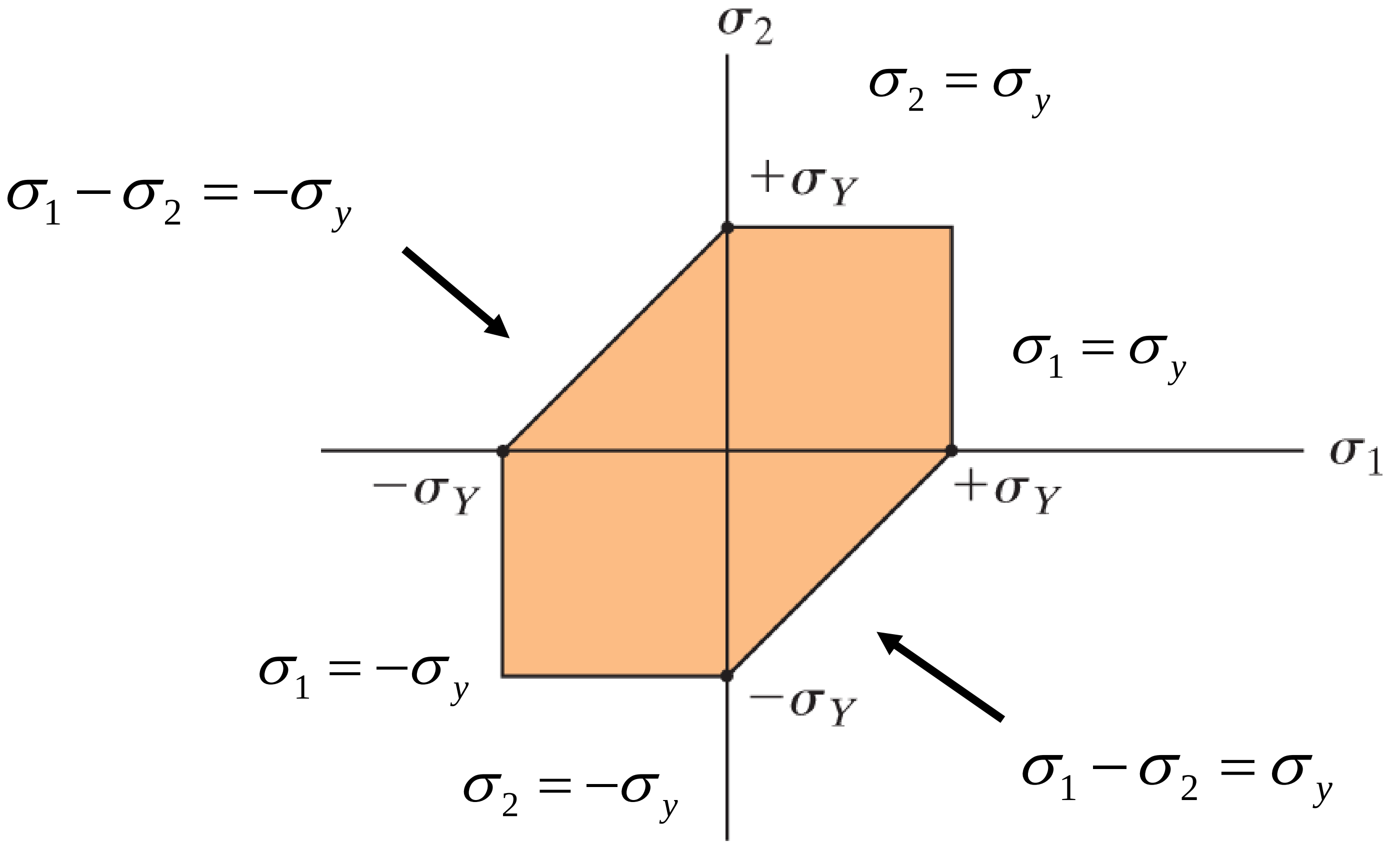
$$\left.\begin{array}{l} |\sigma_1| = \sigma_Y \\ |\sigma_2| = \sigma_Y \end{array}\right\} \sigma_1 \text{ and } \sigma_2 \text{ have the same sign}$$



$$|\sigma_1 - \sigma_2| = \sigma_Y \left.\vphantom{\sigma}\right\} \sigma_1 \text{ and } \sigma_2 \text{ have opposite signs}$$



Maximum-Shear-Stress Theory



Maximum-shear-stress theory

Maximum-Distortion-Energy Theory

strain-energy density

if the material is subjected to a uniaxial stress

$$u = \frac{1}{2}\sigma.\varepsilon$$

If the material is subjected to triaxial stress

$$u = \frac{1}{2}\sigma_1.\varepsilon_1 + \frac{1}{2}\sigma_2.\varepsilon_2 + \frac{1}{2}\sigma_3.\varepsilon_3$$

Maximum-Distortion-Energy Theory

If the material behaves in a linear-elastic manner, then Hooke's law applies

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right) \quad \Rightarrow \quad \underbrace{u = \frac{1}{2}\sigma_1.\varepsilon_1 + \frac{1}{2}\sigma_2.\varepsilon_2 + \frac{1}{2}\sigma_3.\varepsilon_3}$$

$$\varepsilon_z = \frac{1}{E}\left(\sigma_z - \nu\left(\sigma_y + \sigma_x\right)\right)$$

$$\Downarrow$$

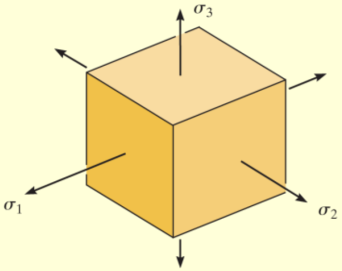
$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\upsilon\left(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_3\sigma_2\right)\right]$$

Maximum-Distortion-Energy Theory

Deform

strain-energy density

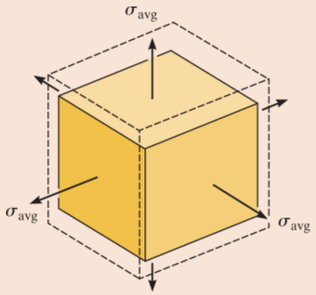
$$u = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\upsilon\left(\sigma_1\sigma_2 + \sigma_1\sigma_3 + \sigma_3\sigma_2\right)\right]$$



Volume Change

energy needed to cause a *volume change* of the element with no change in shape

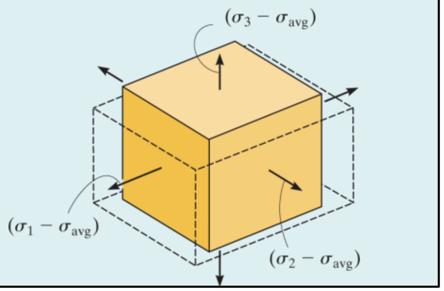
$$\sigma_{avg} = \left(\sigma_1 + \sigma_2 + \sigma_3\right)/2$$



Distort

energy needed to *distort* the element

$$u_d = \frac{1+\upsilon}{6E}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]$$



Maximum-Distortion-Energy Theory

Experimental evidence has shown that materials do not yield when subjected to a uniform stress
As a result, in 1904, M. Huber proposed that yielding in a ductile material occurs when the distortion energy per unit volume of the material equals or exceeds the distortion energy per unit volume of the same material when it is subjected to yielding in a simple tension test.

$$u_d = \frac{1+\upsilon}{6E}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]$$

Maximum-Distortion-Energy Theory

In the case of *plane stress* $\sigma_3 = 0$

$$u_d = \frac{1+\upsilon}{3E}\left[\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right]$$

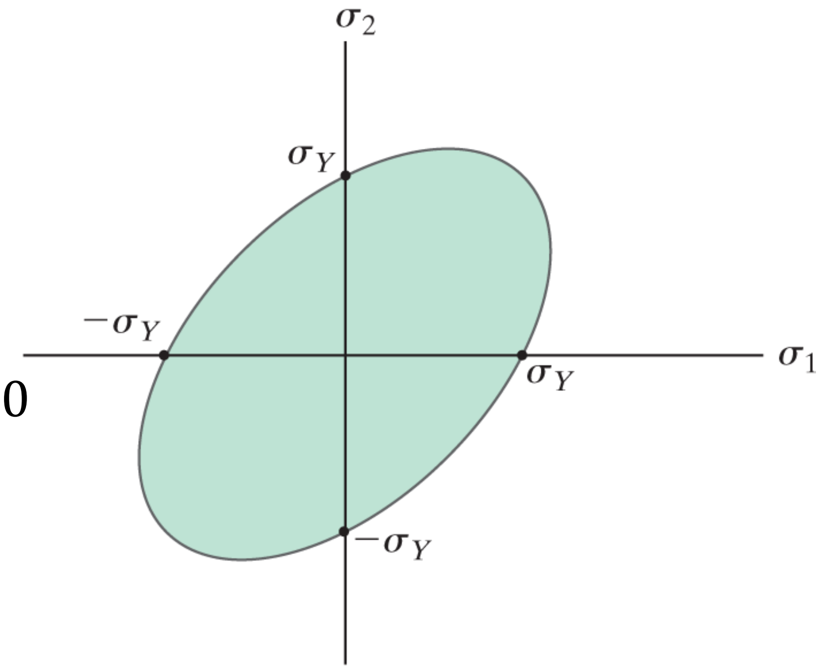
For a *uniaxial* tension test $\sigma_1 = \sigma_Y$ $\sigma_2 = \sigma_3 = 0$

$$(u_d)_Y = \frac{1+\upsilon}{3E}\sigma_Y^2$$

Since the maximum-distortion-energy theory requires

$$\sigma_Y^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$$

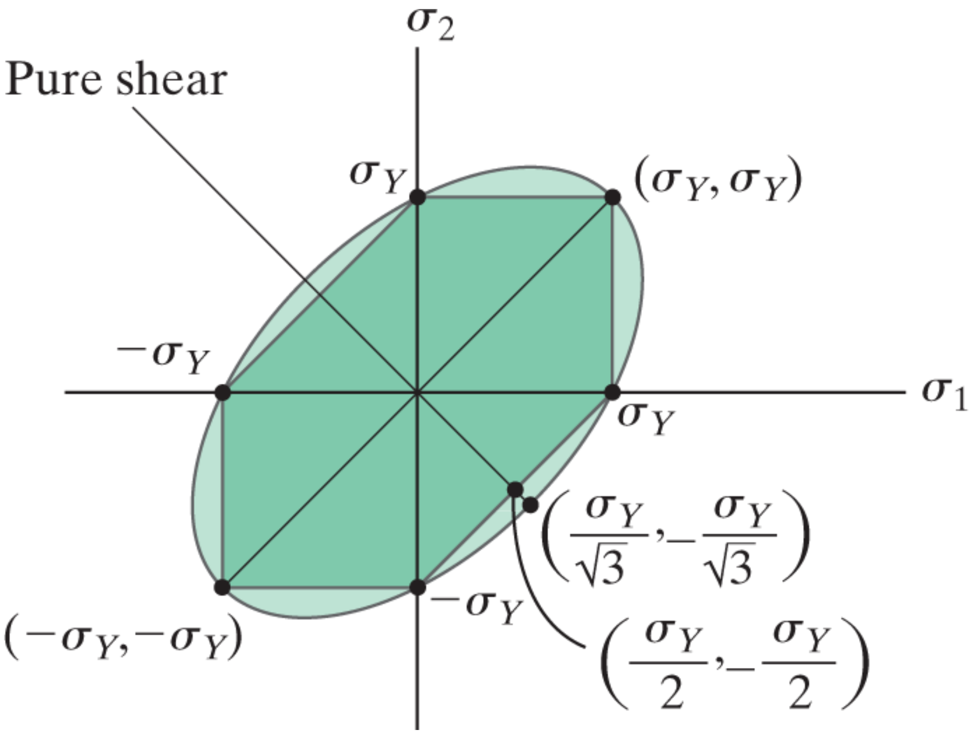
equation of an ellipse



Maximum-distortion-energy theory

Thus, if a point in the material is stressed such that is plotted on the boundary or outside the shaded area, the material is said to fail.

MSS vs MDE



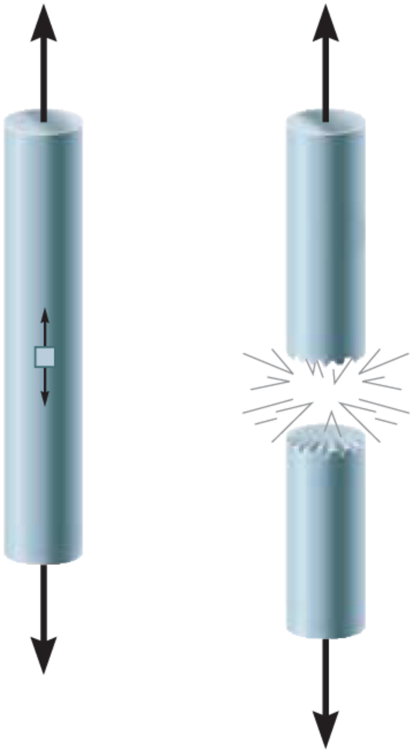
Maximum-Shear-Stress Theory (MSS)

Slightly more conservative
Easier to calculate

Maximum-Distortion-Energy Theory (MDE)

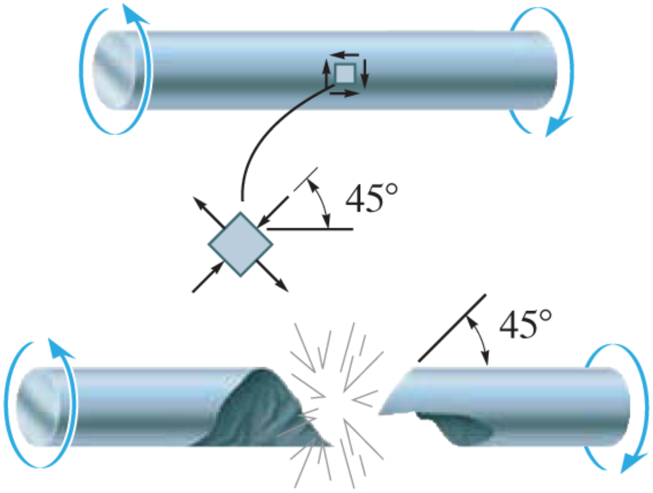
More accurate
If not specified, use this one!

Failure of brittle material



Failure of a brittle material
in tension

The fracture surface is planar



Failure of a brittle material
in torsion

The fracture surface is helical

Maximum-Normal-Stress Theory

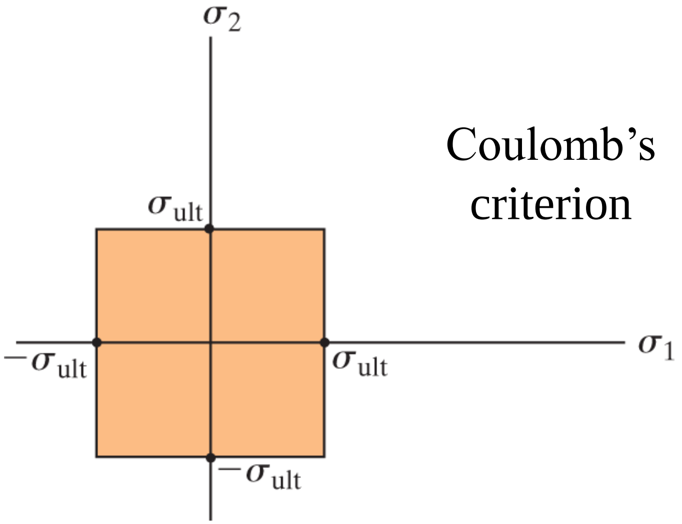
the **maximum-normal-stress theory** states that a brittle material will fail when the maximum tensile stress, σ_1, in the material reaches a value that is equal to the ultimate normal stress the material can sustain when it is subjected to simple tension.

If the material is subjected to plane stress, we require that

$$|\sigma_1| = \sigma_{ult}$$

$$|\sigma_2| = \sigma_{ult}$$

Coulomb's criterion



Maximum-normal-stress theory

Mohr’s Failure Criterion.

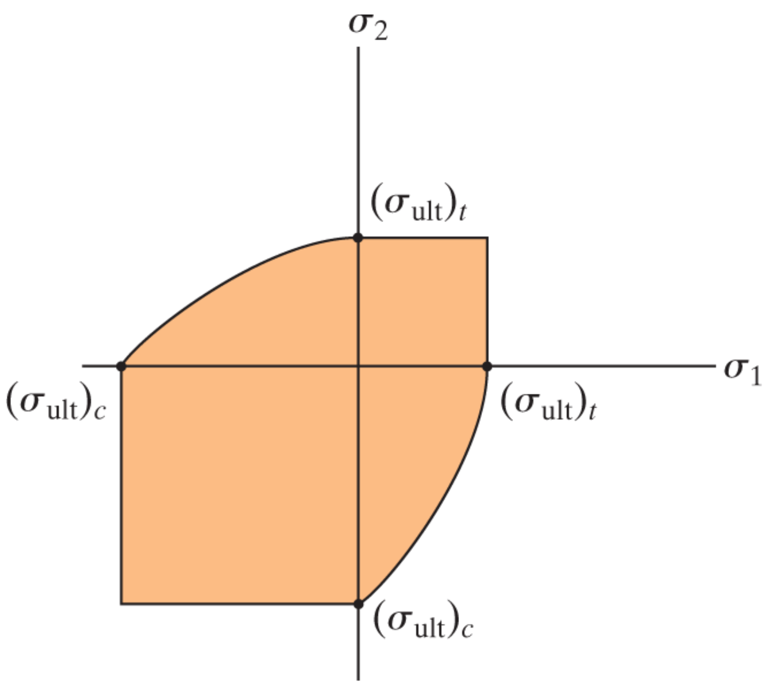
In some brittle materials tension and compression properties are different. When this occurs a criterion based on the use of Mohr’s circle may be used to predict failure

Three tests on the material

A uniaxial tensile test → ultimate tensile stress
A uniaxial compressive test → ultimate compressive stress
A torsion test → ultimate shear stress

Mohr’s Failure Criterion.

Plot Mohr’s circle for each of these stress conditions



Mohr’s failure criterion